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Topics from One-Dimensional Dynamics

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Preface

One-dimensional dynamics owns many deep results and avenues of active mathematical research. Numerous inroads to this research exist for the advanced undergraduate or beginning graduate student. It is precisely these students whom we target. Several glimpses into one-dimensional dynamics are provided with the hope that the results presented illuminate the beauty and excitement of the field. Many topics covered appear nowhere else in "textbook format," some are mini new research topics in themselves, and for nearly all topics we try to provide novel connections with other research areas both inside and outside the text. Among these topics are kneading theory and Hofbauer towers; detailed structure of ω-limit sets; topological entropy; lapnumbers and Markov extensions; the 2^∞ map (Feigenbaum-Coullet-Tresser), interplay amongst continued fractions, adding machines, circle maps, and unimodal maps; irrational rotations as factors of unimodal maps; connections between β-transformations and unimodal maps; Ledrappier's three-dot example; and itineraries for complex quadratic maps and Hubbard trees. The flavor is largely combinatoric, symbolic, and topological. The material presented is *not* meant to be approached in a linear fashion. Rather, we strongly encourage readers to pick and choose topics of interest. Trail routes (other than $n \mapsto n+1$) are indicated in Figure 1; more explicit information is provided at the beginning of each chapter. Suggested uses for the text include: dynamics courses, master theses, reading courses, research experiences for undergraduates (REUs), seminars, senior projects, and summer courses.

As mentioned, the topics covered are *not* the typical topics seen in undergraduate/graduate dynamics texts. Rather, the material is a filtering from the research literature of currently active topics that can be made accessible to the targeted student audience. Frequent references to the literature are made to enable the reader to further investigate the topic under study. Occasionally new or simplified proofs for existing results are given. The level of "mathematical sophistication" required by the reader varies between chapters. The text is also designed to provide the reader with a "critical mass" of results and tools, allowing for further investigation into the field.

Although this text can be used in a "lecture format" course, we have purposely employed a style such that a more active role can be played by students. Exercises are not reserved for the end of a section but rather are sprinkled throughout a section. The student is encouraged to complete exercises as s/he progresses through the material. More challenging exercises are tagged with a ♢. For some exercises, having referenced literature at hand is suggested and such exercises are tagged with a ♣. The Java Applets, `www.uwm.edu/~kmbrucks/Dyntext.html`, assist with the exploration of many topics.

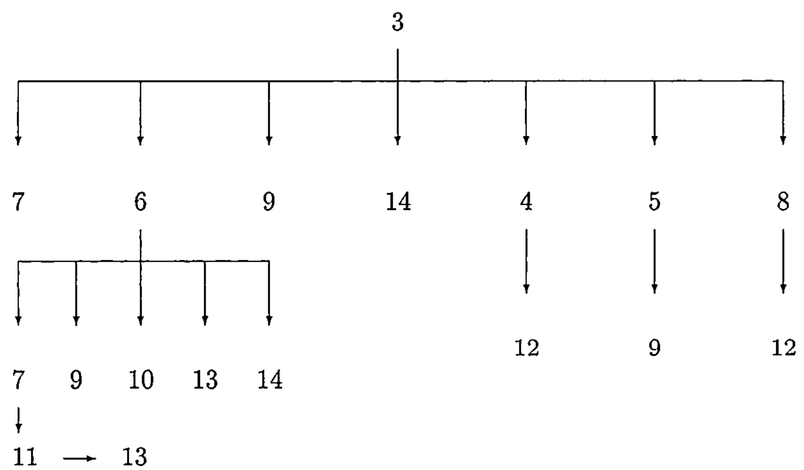

Figure 1: Trail guide

Historically, it was in the 1970s that dynamical systems had a "surge" of activity (which continues today). This surge was due largely to the accessibility of computers and the multitude of results and questions achieved with the aid of computers. However, a genealogical tree of dynamics would include G. D. Birkhoff (1884–1944), M. L. Cartwright (1900–1998), P. Fatou (1878–1929), G. Julia (1893–1978), and H. Poincaré (1854–1912). Historical discussions can be found in [4, 5, 108, 115].

Research in one-dimensional dynamics was ignited in the 1960s and 1970s by American, French, and Ukrainian mathematicians [58, 68, 69, 107, 116, 122, 155, 160]. Metric properties of the Coullet-Tresser (or Feigenbaum) map (see Chapter 5) were studied in the late 1970s [58, 68, 69] and somewhat earlier (and more narrowly) in [127]. The famous theorem of Sharkovsky (see Theorem 3.1.5) appeared in a Ukrainian journal in 1964 [155]. Work of Stefan [160] in 1977 helped bring Sharkovsky's theorem to the west. Around this same time, Li and Yorke published their famous paper "Period three implies chaos" [107]. Published works on the combinatorics of one-dimensional dynamics first appeared in the early 1970s [116, 122]. From these roots, one-dimensional dynamics has flourished into an active and exciting area of mathematical research.

Chapters 1 and 2 provide background material from topology and measure theory, respectively. Rather than begin with these chapters, we suggest the reader refer to these chapters as needed. These chapters are purposely succinct and provide multiple references for the reader looking for more detail/discussion. Lebesgue measure is the only measure used, and it appears infrequently. Hence the reader unfamiliar with measure theory can still access most of the text.

Chapter 3 provides an introduction to symbolic and topological dynamics. This material provides a dynamical foundation for the topics covered in later chapters. We suggest that the reader who is unfamiliar with this material work through the first four sections and move on, returning for the remaining material (recurrence and shift spaces) as needed. A beginning discussion on measurable dynamics is given in Chapter 4. Pieces of this material are referenced in Chapter 12; however, the material is not required for Chapter 12.

The dynamics of a 2^∞ map are investigated in Chapter 5. In particular, the dynamics of this map are shown to be that of the dyadic adding machine. The discussion (in Chapter 5) is accessible to an undergraduate student with minimal experience doing proofs. Generalized versions of the dyadic (triadic, etc.) adding machine are investigated in Chapters 11 and 13.

Chapters 6 and 7 provide combinatoric, number theoretic, and symbolic machinery to investigate one-dimensional dynamical systems. Chapters 8 to 14 target the student with a higher level of mathematical sophistication. Any one of these chapters (8–14) could serve as a component of a dynamics course or as material for a reading course, masters thesis, seminar, summer course, or term project. Of course, this text is not a comprehensive work on one-dimensional dynamics. Other texts in dynamics include [4, 5, 18, 53, 56, 64, 74, 96, 108, 115, 139, 156, 162, 168].

We thank Kurt Cogswell, Beverly Diamond, Jane Hawkins and her students, Chris Sears, and Lubo Snoha for their careful reading of the text; Jane Hawkins for contributing Chapters 2 and 4; Karsten Keller for the figure files, Peter Raith for e-mail discussions on Hofbauer towers; and Christopher Sears for the Java Applets.

This text grew out of courses taught (1997, 1999, 2001) at the Carleton Summer Mathematics Program (SMP) for undergraduate women in mathematics. Each summer, the SMP draws 18 talented undergraduate women

interested in mathematics to Carleton College for four weeks of intensive mathematics study. We applaud Deanna Haunsperger and Steven Kennedy for running a program that so positively impacts all involved.

The first author thanks her family for their ongoing support, which occurs with full measure and densely on her life timeline. The second author was supported by a fellowship of the Royal Netherlands Academy of Sciences and Arts (KNAW). Both authors thank their colleagues and friends for many enjoyable and fruitful communications, all of which make this text possible. Comments are most welcome; send comments to kmbrucks@uwm.edu.