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052154677X - The Cauchy-Schwarz Master Class: An Introduction to the Art of
Mathematical Inequalities

J. Michael Steele

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