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052154677X - The Cauchy-Schwarz Master Class: An Introduction to the Art of
Mathematical Inequalities

J. Michael Steele

Frontmatter

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THE CAUCHY–SCHWARZ MASTER CLASS

This lively, problem-oriented text is designed to coach readers toward mastery of the most fundamental mathematical inequalities. With the Cauchy–Schwarz inequality as the initial guide, the reader is led through a sequence of fascinating problems whose solutions are presented as they might have been discovered — either by one of history’s famous mathematicians or by the reader. The problems emphasize beauty and surprise, but along the way readers will find systematic coverage of the geometry of squares, convexity, the ladder of power means, majorization, Schur convexity, exponential sums, and the inequalities of Hölder, Hilbert, and Hardy.

The text is accessible to anyone who knows calculus and who cares about solving problems. It is well suited to self-study, directed study, or as a supplement to courses in analysis, probability, and combinatorics.

J. Michael Steele is C. F. Koo Professor of Statistics at the Wharton School, University of Pennsylvania. He is the author of more than 100 mathematical publications, including the books *Probability Theory and Combinatorial Optimization* and *Stochastic Calculus and Financial Applications*. He is also the founding editor of the *Annals of Applied Probability*.

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University of Pennsylvania



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Preface

In the fine arts, a master class is a small class where students and coaches work together to support a high level of technical and creative excellence. This book tries to capture the spirit of a master class while providing coaching for readers who want to refine their skills as solvers of problems, especially those problems dealing with mathematical inequalities.

The most important prerequisite for benefiting from this book is the desire to master the craft of discovery and proof. The formal requirements are quite modest. Anyone with a solid course in calculus is well prepared for almost everything to be found here, and perhaps half of the material does not even require calculus. Nevertheless, the book develops many results which are rarely seen, and even experienced readers are likely to find material that is challenging and informative.

With the Cauchy–Schwarz inequality as the initial guide, the reader is led through a sequence of interrelated problems whose solutions are presented as they might have been discovered — either by one of history’s famous mathematicians or by the reader. The problems emphasize beauty and surprise, but along the way one finds systematic coverage of the geometry of squares, convexity, the ladder of power means, majorization, Schur convexity, exponential sums, and all of the so-called classical inequalities, including those of Hölder, Hilbert, and Hardy.

To solve a problem is a very human undertaking, and more than a little mystery remains about how we best guide ourselves to the discovery of original solutions. Still, as George Pólya and others have taught us, there are principles of problem solving. With practice and good coaching we can all improve our skills. Just like singers, actors, or pianists, we have a path toward a deeper mastery of our craft.

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