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0521546192 - Linear Operators and Linear Systems: An Analytical Approach to Control Theory

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JONATHAN R. PARTINGTON

*University of Leeds*



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# Preface

It should be emphasised at the start that this book does not claim to be an exhaustive treatise on either linear operators or linear systems, but it presents an introduction to the common ground between the two subjects, one pure mathematical and one applied, by regarding a linear system as a (causal) shift-invariant operator on a Hilbert space such as  $\ell^2(\mathbb{Z}_+)$  or  $L^2(0, \infty)$ . It therefore includes material on Hardy spaces, shift-invariant operators, the commutant lifting theorem, and almost-periodic functions, which might traditionally be regarded as “pure” mathematics, and is suitable for those working in analysis who wish to learn more advanced material on linear operators.

At the same time, it is hoped that students and researchers in systems and control will find the approach taken attractive, including as it does much recent material on the mathematical side of systems theory, which cannot easily be found elsewhere: these include recent developments in robust control, power signal spaces, and the input–output approach to time-delay systems. Parts of this book have been expounded in graduate courses and other lectures at that level and could be used for a similar purpose elsewhere.

Chapter 1 begins with a review of basic operator theory without proofs. All this material can be found in any introductory course and many textbooks, and so is included mostly for reference. The other main topic of this chapter, which is treated in considerably more detail, is that of Hardy spaces, which are Banach spaces of analytic functions on the disc or half-plane. Our treatment covers the essential ideas (in particular inner and outer functions) that will be needed later.

In Chapter 2 we begin with material that will be unfamiliar to many readers, namely, the study of unbounded closed operators. The approach is to study an operator by means of its graph, and we also introduce semigroups and the gap metric. Most of the material is fairly standard, although its approach is slanted towards the applications to be encountered later. We also include a brief discussion of admissibility, a topic of active research that has not yet reached the standard textbooks.

Chapter 3 establishes the link between linear systems and shift-invariant operators. The standard Beurling–Helson–Lax–Wiener theorems on invariant subspaces are presented, with the most elementary proofs available; then we go on to shift-invariant operators, which can be thought of as operators with shift-invariant graphs. The other property that distinguishes a linear system is the idea of causality, and we include a discussion of the now-notorious Georgiou–Smith paradox. We conclude with a gentle treatment of the commutant lifting theorem, an abstract theorem in operator theory with many attractive applications.

Chapter 4 brings together the ideas of the previous chapter to discuss robust control (i.e., stabilization by feedback) from a graph point of view using coprime factorizations. This is by no means a new idea, but the mathematical developments of the 1990s that link it with the idea of shift-invariance have put this on a much more rigorous footing, and it is time to present the material in a more operator-theoretic way. We also discuss the chordal metric, originating in complex analysis, which under the name “pointwise gap metric” provides another useful way of measuring the distance between linear systems.

Chapter 5 presents the topic of spaces of persistent signals. These interest engineers greatly, but the subject is a minefield, in that several errors are reproduced in the literature. We start conventionally with an easy introduction to almost-periodic functions, presented as the simplest persistent signals; there are many accounts of these, but no recent ones seem to be as clear as the original ones of Bohr and Besicovitch; these in turn suffer from using what is now a rather old-fashioned notation. After that we move on to bounded-power signals, where we draw largely on papers written within the last five years. We conclude with non-stochastic approaches to white noise, correlation, and parts of Wiener’s generalized harmonic analysis, all treated from an operator-theory viewpoint.

Chapter 6 begins with a brief discussion of finite-dimensional systems, but the main topic is that of delay systems, which we may think of as the simplest and most important example of an infinite-dimensional system, having the greatest interest from an analytic point of view. The four themes of this chapter are the classification of delay systems into retarded, neutral and advanced types; stability, a question that resolves itself into asking when delay systems represent operators of multiplication by bounded functions, and how to locate the poles of their transfer functions; rational approximation, which can now be presented in an elementary fashion using shift operators; and finally stabilization, where we apply the ideas of Chapter 4 to this concrete situation. In all cases we give a presentation with the minimum of unnecessary technical detail.

The book includes approximately 100 exercises, which are mostly intended to

be quite easy and to give further practical illustrations of the main results in the text.

Since this book's theme is the link between operator theory and systems theory, we deliberately omit certain topics that do not fit into this approach, for example, the spectral theory of non-compact operators. The reader wishing to learn more advanced functional analysis and operator theory is recommended to consult the books [25, 67, 117, 146], for example. Likewise, most treatments of systems theory tend to concentrate on the finite-dimensional case and then go into more details of control design. Of these, the texts [37, 49, 66, 75, 132] may be recommended.

We take the opportunity to mention some other interesting books that take an operator-theoretic approach to linear systems, for example, [31, 32, 38]. These seem to be pitched at a more advanced level, and their selection of material is rather different. This last comment is appropriate for the monographs [20, 151], which we may expect to become classics.

I am grateful to Birgit Jacob, Romesh Kumar, Pertti Mäkilä, Denis Matignon and Martin Smith for their helpful comments on the first draft of this manuscript. Some of the book was written when I was visiting the University of Lyon I, and I thank Isabelle Chalendar and Monique Gaffier for their help in making this possible. I also wish to thank Roger Astley and Elise Oranges for their advice on the production of this book. Finally, this book is dedicated affectionately to Andrew, Chris and Kate, who between them have put up with me for more than a century.