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Transcendental Aspects of Algebraic Cycles

Proceedings of the Grenoble Summer School, 2001

Edited by

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and

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Preface

General facts

The Institut Fourier has been organizing summer schools in mathematics for several years. They are intended for researchers as well as graduate students from all countries. One of the main aims is to transmit fundamental knowledge in the field of mathematics and to promote the exchange of ideas between researchers. The subject of each summer school is closely linked with the research themes of the Institut Fourier's teams. During these schools, several foreign and French lecturers are invited. The graduate students will get an efficient and useful complementary training and have an opportunity to get in touch with the most current research in the world. The participants will discover the Grenoble department and may pursue their research in collaboration with Grenoble researchers.

Each summer school takes place at the Institut Fourier, lasts three weeks and hosts about 70 participants (upon application, a selection will be made). Twenty to 25 hours of lectures or seminars are given each week on different themes.

The subject of the Summer School 2001 was 'Transcendental Aspects of Algebraic Cycles'.

Organizers: Chris Peters (Institut Fourier, Grenoble) and Stefan Müller-Stach (University Essen, Germany).

About these proceedings

Introductory material

The first week of the Summer School was devoted to explaining some basic material in order to make up for the different backgrounds and levels of the participants.

Firstly Stefan Müller-Stach explained Griffiths' theory of the period map starting from elliptic curves, moving up to higher genus curves and then to the general situation of a family of projective varieties. This was preceded by a users guide to the de Rham, singular and simplicial (co)homology presented by Chris Peters.

We decided not to publish our notes from these two lecture series in the current Proceedings since they form the beginning of a book by Jim Carlson and the editors of these proceedings (see [1]). For this there are also alternative sources: the book [2] by Clemens treats periods of elliptic curves, the first chapters of the book [4] by Griffiths and Harris can serve as an initiation to cohomological methods in algebraic geometry and finally, for period maps the readers can consult the original papers [5, 6]

Secondly, Javier Elizondo presented a series of lectures on algebraic cycles, their equivalence relations, and Chow varieties. He started out by explaining the various ways of assigning intersection multiplicities to the components of a set-theoretic intersection. He then went on to treat the most important equivalence relations among cycles and showed how this leads to an intersection ring in the case of non-singular quasi-projective varieties. All this foundation material can be found in [3] and is therefore not published in these Proceedings. Instead of equivalence classes of cycles, one can also consider cycles of a fixed dimension and fixed degree (in some projective embedding). This is the set underlying a Chow variety, as next explained by Elizondo in his contribution published here. Chow varieties become the underlying moduli spaces for cycles in Lawson homology. One can combine the topological Euler characteristics of the components of the Chow varieties in a formal series, the Euler–Chow function of the variety. This series has been computed in a number of cases which are explained in detail in Elizondo's notes.

Finally, Siegmund Kosarew and Chris Peters gave an introduction to Lawson homology for projective varieties. Classically one introduces flexibility in the study of cycles by introducing suitable equivalence relations. In Lawson homology this flexibility is created by looking at the homotopy type of cycle spaces, i.e. the group of cycles equipped with the Chow topology coming from the topology on the Chow varieties. The contribution of Kosarew and Peters starts out by recalling the basics from homotopy theory. It is followed by one of the definitions of Lawson homology. Since functoriality can only be understood in the framework of continuous algebraic maps, this notion is explained next. This is followed by a discussion of the basic ingredient in the theory, the Chow topology. The article ends with various other useful definitions of Lawson homology which are shown to be equivalent. This is based on some basic but sophisticated simplicial constructions which the authors only sketch but for which references are given.

Lawson (co)homology

Paulo Lima-Filho gave a series of lectures treating various foundational aspects of Lawson homology and cohomology for algebraic varieties, not just for projective varieties.

An axiomatic set-up of Chow topology is at the basis of Lima-Filho's treatment which he presents in the first lecture. He also treats the cycle class maps which compare Lawson homology to ordinary singular homology and the s-maps which link Lawson homology groups mapping to the same homology group under the class map.

One cannot directly put a mixed Hodge structure on Lawson homology, but one has to do this through colimits as explained in the second lecture.

Although there is no good intersection theory for cycles, there is one at the level of homotopy groups, i.e. the Lawson homology groups. This, as well as morphic cohomology, the cohomological counterpart of Lawson homology, is explained in the final lecture.

Motives and motivic cohomology

In Jacob Murre's article, pure motives in the sense of Grothendieck are discussed, i.e. motives arising from smooth, projective varieties. Here, first the categories of pure motives are defined using correspondences and a suitable equivalence relation. This construction does not depend on any conjectures. However, the properties of these categories depend very much on the so-called standard conjectures. This is discussed in detail in Murre's lectures. Further topics are: Manin's identity principle, Jannsen's theorem on motives modulo numerical equivalence and conjectural filtrations on Chow groups arising from Chow–Künneth decompositions of the diagonal. The article contains many illustrating examples, such as the motive of curves, surfaces, Albanese and Picard motives and Chow–Künneth decompositions of elliptic modular varieties.

The article by Philippe Elbaz-Vincent consists of a concise approach to motivic cohomology via higher Chow groups. Both concepts are known to be the same by a theorem of Voevodsky. Historically, higher Chow groups were the first candidate for motivic cohomology. These were defined by Bloch around 1985. Elbaz-Vincent first summarizes the basic functorial properties of higher Chow groups including localization and the Mayer–Vietoris property. Then he discusses in detail higher Chow groups of smooth local rings and the

Gersten conjecture, which leads to a local-to-global principle for motivic cohomology. Also the Bloch–Lichtenbaum–Levine–Friedlander–Suslin spectral sequence from higher Chow groups to algebraic K-theory is presented. At the end of this paper many examples and applications of the previous results are explained.

Hodge theoretic invariants of cycles

In James Lewis' 'Three lectures on the Hodge conjecture', the Hodge conjecture, in its classical form as well as in the general (Grothendieck's amended) form are presented. The topological cycle class map is revisited and the classical Hodge conjecture is first discussed together with several examples, including uniruled fourfolds after Conte and Murre. Lefschetz' original approach using normal functions is also treated, culminating in Zucker's theorem on the cubic fourfold. However, owing to failure of Jacobi inversion this approach is very limited. In his second lecture, Lewis explains the geometric approach using cylinder maps. This proves the Hodge conjecture for many types of hypersurfaces like the quintic fourfold. Finally, he discusses the approach going back to Colliot-Thélène, Bloch and Srinivas where a decomposition of the diagonal in $X \times X$ is used. All explanations are covered by many examples and additional related material.

Jan Nagel presents the foundations of some infinitesimal Hodge theoretic methods going back to Griffiths. These can be applied to study algebraic cycles. He first explains the concept of normal functions by introducing all the complex differential geometry around families of algebraic cycles and their Abel–Jacobi classes living in families of complex tori. The fundamental concept here is the infinitesimal invariant of a normal function, first defined by Griffiths and later refined by Green and Voisin. He proceeds to give a proof of the original theorem of Griffiths about what is now called the Griffiths' groups. In Section 7.3, the theorem of Green and Voisin on the Abel–Jacobi map for hypersurfaces of large degrees is discussed and proved. This uses Koszul theoretic and monodromy methods. Finally, an effective version of Nori's connectivity theorem and a sketch of its proof is presented together with a nice set of applications, including higher Chow groups of certain complete intersections of large degree.

The article 'Beilinson's Hodge and Tate conjectures' by Shuji Saito is devoted to an application of Hodge theoretic invariants of non-compact varieties to the study of algebraic cycles. He reviews his results with Asakura on the computation of Gauss–Manin complexes on families of open complete intersections via a generalization of the Dwork–Griffiths methods of residues. This leads to a proof of the Beilinson Hodge and Tate conjectures in several cases and has applications to the injectivity of cycle class maps for higher Chow groups which are also discussed. The paper finishes with an application to a Noether–Lefschetz problem for K_2 of open surfaces. This shows once again the strength of Hodge theoretic methods applied to interesting mathematical problems.

Schedule of the summer school

First week: June 18–June 22, 2001

Lecturers: J. Elizondo (JE), S. Kosarew (SK), S. Müller-Stach (SM), J. Nagel (JN), C. Peters (CP)

Day	Hour	Speaker	Title
Monday June 18	08:00–09:00		*Welcome*
	09:00–10:30	CP	Cohomology of compact Kähler manifolds (1)
	10:30–11:00		*Coffee break*
	11:00–12:30	JE	Algebraic cycles, moving lemma, intersection theory
	14:00–15:00	CP	Cohomology of compact Kähler manifolds (2)
	15:00–16:00	SM	Periods of elliptic curves (1)
Tuesday June 19	08:30–10:00	CP	Holomorphic invariants (1)
	10:00–10:30		*Coffee break*
	10:30–12:00	SM	Periods of elliptic curves (2)
	13:30–14:30	CP	Holomorphic invariants (2)
	14:30–15:30	JE	Equivalence relations of cycles
Wednesday June 20	08:30–10:00	SM	Period maps (1)
	10:00–10:30		*Coffee break*
	10:30–12:00	JE	Chow varieties (1)
	13:30–14:30	SM	Period maps (2)
	14:30–15:30	CP	Cycle spaces
Thursday June 21	08:30–10:00	CP	Lawson homology: introduction (1)
	10:00–10:30		*Coffee break*
	10:30–12:00	SM	Infinitesimal period maps
	13:30–14:30	JN	Normal functions
	14:30–15:30	SK	Lawson homology: introduction (2)
Friday June 22	08:30–10:00	SK	Lawson homology: introduction (3)
	10:00–10:30		*Coffee break*
	10:30–12:00	JN	Infinitesimal invariants of normal functions
	13:30–14:30	JN	The theorem of Green–Voisin
	14:30–15:30	JE	Chow varieties (2)

Second week: June 25–June 29, 2001

Lecturers: A. Collino (AC), P. Elbaz-Vincent (PE), J. Elizondo (JE), J. Lewis (JL), P. Lima-Filho (PL), J. Murre (JM), J. Nagel (JN)

Day	Hour	Speaker	Title
Monday June 25	08:30–10:00	JN	The effective Nori theorem (1)
	10:00–10:30		*Coffee break*
	10:30–12:00	PE	Higher Chow theory (1)
	13:30–14:30	AC	Abel–Jacobi maps (1)
	14:30–15:30	JM	Motives (1)
Tuesday June 26	08:30–10:00	JM	Motives (2)
	10:00–10:30		*Coffee break*
	10:30–12:00	AC	Abel–Jacobi maps (2)
	13:30–14:30	PE	Higher Chow theory (2)
	14:30–15:30	JN	The effective Nori theorem (2)
Wednesday June 27	08:30–10:00	JM	Motives (3)
	10:00–10:30		*Coffee break*
	10:30–12:00	JL	The Hodge conjecture (1)
	13:30–14:30	AC	Abel–Jacobi maps (3)
	14:30–15:30	PL	Lawson homology (1)
Thursday June 28	08:30–10:00	JN	The effective Nori theorem (3)
	10:00–10:30		*Coffee break*
	10:30–12:00	PL	Lawson homology (2)
	13:30–14:30	PE	Higher Chow theory (3)
	14:30–15:30	JL	The Hodge conjecture (2)
Friday June 29	08:30–10:00	JL	The Hodge conjecture (3)
	10:00–10:30		*Coffee break*
	10:30–12:00	PL	Lawson homology (3)
	13:30–14:30	PL	Lawson homology (4)
	14:30–15:30	JE	Euler–Chow series

Third week: July 2–July 6, 2001
Lecturers: E. Mouroukos (EM), A. Reznikov (AR), S. Saito (SS)

Day	Hour	Speaker	Title
Monday July 2	08:30–10:00	SS	Applications of Hodge theory to algebraic cycles (1)
	10:30–12:00	EM	Motives (4)
Tuesday July 3	10:30–12:00	EM	Motives (5)
Wednesday July 4	08:30–10:00	AR	Bloch's conjecture (1)
	10:30–12:00	SS	Applications of Hodge theory to algebraic cycles (2)
Thursday July 5	08:30–10:00	AR	Bloch's conjecture (2)
	10:30–12:00	EM	Motives (6)
Friday July 6	08:30–10:00	AR	Work of Bloch–Esnault, Esnault–Srinivas
	10:30–12:00	SS	Applications of Hodge theory to algebraic cycles (3)

Invited speakers: S. Archava (SA), M. Asakura (MA), E. Colombo (EC), P. Del Angel (PD), R. Joshua (RJ), M. Kerr (MK), R. Laterveer (RL), A. Otwinowska(AO), O. Penacchio (OP), P. F. dos Santos (PdS)

Day	Hour	Speaker	Title
Monday July 2	13:30–14:30	MA	On the regulator image of K_2
	14:30–15:30	RL	Relative version of Mumford's theorem on 0-cycles
Tuesday July 3	08:30–10:00	RJ	Applications of the strong Künneth decomposition of the diagonal
	13:30–14:30	EC	Mixed Hodge structures on the fundamental group of a hyperelliptic curve and algebraic cycles
	14:30–15:30	SA	Arithmetic Hodge structures on homotopy groups and intersection theory of algebraic cycles
Wednesday July 4	13:30–14:30	PD	K-theory and the Painlevé equation
	14:30–15:30	PdS	Lawson homology for real varieties
Thursday July 5	13:30–14:30	JM	Chow–Künneth decomposition for elliptic modular varieties
	14:30–15:30	AO	Hodge locus of a family of hypersurfaces
Friday July 6	13:30–14:30	MK	Milnor regulators and higher Abel–Jacobi maps
	14:30–15:30	OP	The R-split level of variations of mixed Hodge structures

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[4] Griffiths, P. and Harris, J. *Principles of Algebraic Geometry*, John Wiley, New York, 1978.

[5] Griffiths, P. Periods of integrals on algebraic manifolds I, II, *Am. J. Math*. **90** (1968) 568–626, 805–865.

[6] Griffiths, P. Periods of rational integrals on algebraic manifolds III, *Publ. Math. Inst. Hautes Etudes. Sci.* **38** (1970) 125–180.

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