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Preface

In 1999 a number of eminent mathematicians were invited to Bielefeld, to present papers at a one-week conference devoted to interactions between (mostly) infinite groups on the one hand and topological, combinatorial and arithmetic structure on the other. The present volume consists of articles invited from participants in this conference.

A glance at the table of contents gives an immediate impression of the breadth and depth of the contributions included here. The study of topological finiteness properties, a beautiful field of research inhabiting the fertile region between group theory and geometry, is the subject matter of papers by Abramenko, Behr, and Bux, while the article by Bieri and Geoghegan extends this theme towards a theory of group actions on non-positively curved spaces. Another exciting topic of somewhat similar geometric flavour is the theory of Kac-Moody groups, of which Remy's article gives a timely and masterly exposition, incorporating much of his own research. The paper by Chiswell, almost a monograph on its own provides a surprisingly accessible and highly readable account of the theory of Euler characteristics, an account which had been sorely missed in the literature.

The papers by Nekrashevych and Sidki and by Parker and Series both explore the fruitful connection between groups and inherent geometric structure on the one hand, and formal languages and automata on the other. In recent years, automorphisms of regular 1-rooted trees of finite valency have been the subject of vigorous research as a source of remarkable groups, whose structure reflects the recursiveness of these trees; cf. for instance the article by Sergiescu surveying the construction due to Gupta and Sidki of infinite Burnside groups (in: Group Theory from a Geometrical Viewpoint, World Scientific, 1991). This recursiveness in turn can be interpreted in terms of automata; in fact, each automorphism of the tree has a natural interpretation as input-output automaton, where the states, finite or infinite in number, are themselves automorphisms of the tree. This is the context of the article by Nekrashevych and Sidki, which presents a penetrating and original study of state-closed groups of automorphisms of the binary tree, with emphasis on m-dimensional affine groups, that is, groups of the form $\mathbb{Z}^m \cdot \mathrm{GL}(m, \mathbb{Z})$.

For a closed 2-manifold M, the mapping class group $\mathcal{M}(M)$ is defined as the group of all autohomomorphisms of M modulo the subgroup of those deformable to the identity. By a theorem of Nielsen (Acta Math. 50, 189–358), $\mathcal{M}(M)$ is isomorphic to $\mathrm{Out}(\pi_1(M))$, the group of all automorphisms of the fundamental group $\pi_1(M)$ of M modulo inner automorphisms. This important result opens up the possibility of an algebraic approach to the investigation of mapping class groups, which until recently was standard. By way of contrast, the fundamental paper by Parker and Series in this volume studies the mapping class group of the twice punctured torus Σ_2 from a topological and dynamical point of view. Their approach rests on the analogy between Fuchsian groups acting on the hyperbolic plane and the mapping class group $\mathcal{M}(\Sigma_2)$ acting on the corresponding Teichmüller space $\mathcal{T}(\Sigma_2)$. In this analogy, the boundary S^1 of the hyperbolic plane is replaced by the Thurston boundary of $\mathcal{T}(\Sigma_2)$, the space $\mathcal{PML}(\Sigma_2)$ of projective measured laminations on Σ_2, on which $\mathcal{M}(\Sigma_2)$ also acts. There is a well-known relationship between the modular group $\mathrm{PSL}(2, \mathbb{Z})$, thought of as a Fuchsian group acting on hyperbolic 2-space, and continued fractions, thought of as points of the limit set of $\mathrm{PSL}(2, \mathbb{Z})$. The celebrated Bowen-Series construction generalizes this relationship to a large class of Fuchsian groups Γ; it gives rise to a Markov map defined on the boundary at infinity (that is, the limit set $\Lambda(\Gamma)$ of Γ), generating continued fraction expansions for points in $\Lambda(\Gamma)$, whose admissible sequences simultaneously give an elegant solution to the word problem in Γ. A crucial step in the authors' program is the construction of an analogous Markov map on $\mathcal{PML}(\Sigma_2)$, which has the same relation to the action of $\mathcal{M}(\Sigma_2)$ on $\mathcal{PML}(\Sigma_2)$ as the Bowen-Series map has to the action of Γ on $\Lambda(\Gamma) = S^1$. The construction of this Markov map in turn gives rise to an explicit automatic structure on $\mathcal{M}(\Sigma_2)$ in the sense of Epstein et al. In order to illustrate their ideas and techniques, Parker and Series begin by discussing in some detail the (well-known) situation for the once punctured torus, explaining along the way relevant background material and important definitions. Having given this illuminating example, each step is then generalized to the much more demanding situation for Σ_2. While the authors concentrate on the case of the twice punctured torus, their methods in principle appear general and powerful enough to tackle even more advanced situations, and one can hope for important future research along similar lines.

Combinatorial group theory in the classical spirit of Nielsen and Magnus is the context for Collins' contribution; it combines a careful and clear introduction to Magnus' method of investigation for one-relator groups with the solution of a long-standing problem in the field. Situated in the same area, but in a rather different spirit, the paper by Grigorchuk and Wilson explores the intimate connection between atomic groups (in the sense of Pride's pre-order on groups) and just infinite groups. Their main result provides a sufficient condition for branch groups to be atomic, leading to a number of beautiful and highly non-trivial new examples of atomic groups.

In a lecture at Oberwolfach in the 1980s, Helmut Wielandt defended the proposition that topology in permutation groups is of no use, and came to the conclusion that (with the addition of the word "almost") this was indeed the case. Starting with an account of Wielandt's own work on the subject (the connection of non-Hausdorff topologies preserved by a permutation group and the concepts of primitivity and strong primitivity), Cameron's article sets out to qualify this verdict. He gives a detailed account of a spectacular theorem by Macpherson and Praeger, according to which a primitive group preserving no non-trivial topology is highly transitive. The original proof uses substantial machinery from model theory, and Cameron provides a new argument, subtle but essentially elementary, to replace part of their proof. Other topics include Peter Neumann's work on automorphism groups of filters, Mekler's theorem providing a necessary and sufficient condition for a countable permutation group to act on the rationals by homeomorphisms, and the natural topology (of pointwise convergence) on permutation groups.

The theory of subgroup growth, an exciting and fast developing part of what has become known in recent years as 'asymptotic group theory', studies the number theory of arithmetic functions counting (various types of) finite index subgroups in a group, and the connection of such arithmetic information with the algebraic structure of the underlying group. This theory has grown, over the last two decades, out of the work of Grunewald, Lubotzky, Mann, Segal, and others including the present editor, and has already given rise to a number of spectacular results, most of which are described in the recent book by Lubotzky and Segal (Birkhäuser, 2003). The papers by Mann, Müller, and Segal all deal with various aspects of this theory: the article by Avinoam Mann explores the use of probabilistic arguments to prove assertions concerning the subgroup growth of profinite groups; Müller's paper derives detailed congruences for the number of index n subgroups and the number of free subgroups of given index in Hecke groups, that is, groups of the form $C_2 * C_q$ for some $q \geq 3$. The results and methods of the latter paper have opened up a new and thriving chapter of subgroup growth theory dealing with modular properties of subgroup counting functions. Segal's article revolves around the so-called *Gap Conjecture*, which is concerned with the question whether, for finitely generated groups, there is a gap in the possible types of subgroup growth just above polynomial growth. His main result constructs counter-examples to this conjecture possessing a number of remarkable additional properties.

The three articles by Abels, Helling, and Vinberg and Kaplinsky deal with various arithmetic aspects of groups. Hyperbolic lattices in dimension 3, that is, discrete cofinite subgroups of SL(2, $\mathbb{C}$), appear to exhibit the tendency to have integer-valued character functions. Indeed, until recently, the only example in the literature of a lattice with non-integral character occurred in Vinberg's seminal 1967 paper on discrete hyperbolic groups generated by reflections. The article by Helling starts out with a discussion in geometric language of Vinberg's example, complementing Vinberg's results by determining the trace field of this lattice. Helling's main contribution is the construction of a whole series of cocompact lattices, which appear as the result of Dehn surgery along the figure eight knot, at the same time providing strong evidence that an infinite sub-series of these lattices should not admit an integer-valued character. This paper represents a major advance in the study of what appears to be a very difficult problem, and is likely to inspire further research.

By a generalized triangle group one means a group Γ with a fixed presentation of the form

$$\Gamma = \langle x, y \mid x^k = y^\ell = \big(w(x, y)\big)^m = 1\rangle,$$

where $k, \ell, m \geq 2$ and $w(x, y)$ is a word of the form

$$w(x, y) = x^{k_1} y^{\ell_1} x^{k_2} y^{\ell_2} \cdots x^{k_s} y^{\ell_s}$$

with $s \geq 1$, $0 < k_j < k$, and $0 < \ell_j < \ell$. It is also required that $w(x, y)$ is not itself a power of a shorter word. Since their introduction around 1987 by Baumslag, Morgan, Shalen and others, generalized triangle groups have been the object of intense research. One of the main tools in their study are so-called essential homomorphisms, that is, homomorphisms $\rho : \Gamma \to \mathrm{PSL}(2, \mathbb{C})$ such that

$$|\rho(x)| = k, \quad |\rho(y)| = \ell, \quad \text{and} \quad |\rho(w(x, y))| = m.$$

Generalized triangle groups are known to admit such an essential homomorphism to PSL(2, $\mathbb{C}$), and, for most Γ, the image of ρ is in fact infinite. There are, however, infinite generalized triangle groups all of whose essential images are finite. These are the so-called pseudo-finite generalized triangle groups, whose classification appears to be difficult, and is one of the major open problems in the field. The paper by Vinberg and Kaplinsky in this volume represents a substantial advance in this direction; making use of essential homomorphisms in sufficiently large finite cyclic groups as well as methods from number theory, the authors completely cover the cases $m \geq 3$ and $s \leq 3$. Despite these impressive results however, the problem to classify pseudo-finite generalized triangle groups still has to be considered wide open; but ideas and methods employed in this paper will, it is hoped, turn out to be useful in the study of other aspects of this attractive and important problem.

Let G be a reductive group over a local field, Γ an S-arithmetic subgroup of G, and let $\mathfrak{S}$ be a fundamental domain for Γ in G (a so-called Siegel domain). In his 1959 Japan lectures on reduction theory, Carl Ludwig Siegel asked, whether, in modern terminology, the natural map $\mathfrak{S} \to \Gamma\backslash G$ is a coarse isometry. More precisely, Siegel asked this question only for the case where $G = \mathrm{SL}(n, \mathbb{R})$, $\Gamma = \mathrm{SL}(n, \mathbb{Z})$, and with respect to the pseudo-metric on G coming from the standard Riemannian metric on the symmetric space of G (the space of positive-definite real symmetric $n \times n$-matrices). Joint work of Abels and Margulis has recently led to a positive answer in full generality, for arbitrary reductive groups G, S-arithmetic subgroups Γ, and for pseudo-metrics on G which are norm-like. Here, a pseudo-metric is called norm-like, if it is coarsely isometric to a metric coming from the operator norm of a rational representation, or, equivalently, coming from a norm on a maximal split torus. This spectacular result in turn leads to the question of determining which pseudo-metrics are norm-like. The paper by Abels contains foundational material on these two topics. More specifically, it provides four descriptions of one and the same quasi-isometry class of pseudo-metrics on a reductive group G over a local field: the word metric corresponding to a compact set of generators of G, the pseudo-metric given by the isometric action of G on a metric space, the pseudo-metric coming from the operator norm for a representation of G, and the pseudo-metric given on a split torus over a local field K by valuations of the K^*-factors. As a result, one has four intriguingly different descriptions of one and the same natural and distinguished quasi-isometry class of pseudo-metrics. This paper will be indispensable for everyone wishing to study in depth the work of Abels and Margulis, as well as that of other authors, on Siegel's fundamental problem.

When setting out (rather naively) in 1998 to organize the conference which forms the background for this volume, and to edit resulting articles, little did I foresee how formidable and time consuming this task would turn out to be (a feeling no doubt shared by most inexperienced editors). The aim was to produce a stimulating book full of ideas, but also introducing sound technique, comprehensible to good students, which would open up future development in each of the various areas under discussion. All in all, the outcome, in my opinion, surpasses even these rather ambitious expectations; and, in retrospect, I find that I have learned immensely, and have thoroughly enjoyed this task. My gratitude extends to the individual authors; each of them has shared exciting ideas and important insights, but has also made a considerable effort to communicate them. Thanks are also due to Heinz Helling and Barbara Schulten for help with the organization and running of the original conference, and to Jan-Christoph Puchta for conveying to me some of his advanced TeX wisdom.