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0521539579 - Geometric Mechanics and Symmetry: The Peyresq Lectures

Edited by James Montaldi and Tudor Ratiu

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Edited by

James Montaldi
University of Manchester

Tudor Ratiu
Ecole Polytechnique Fédérale de Lausanne



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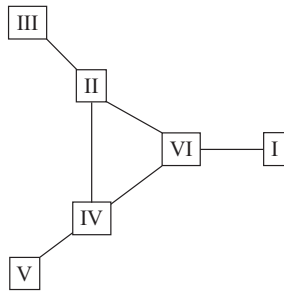
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Contributors

Henk Broer, Department of Mathematics, University of Groningen, P.O. Box 800, 9700 AV Groningen, The Netherlands.

Pietro-Luciano Buono, Faculty of Science, University of Ontario Institute of Technology, 2000 Simcoe St. North, Oshawa, ON L1H 7K4, Canada.

Richard Cushman, Mathematics Institute, University of Utrecht, Budapestlaan 6, 3508TA Utrecht, The Netherlands.

Maria-Cristina Ciocci, Department of Pure Mathematics and Computer Algebra, Ghent University, Krijgslaan 281, 9000 Gent, The Netherlands.

Konstantinos Efstathiou, Department of Physics, University of Athens, Panepistimiopolis Zografos, Athens, Greece.

Frédéric Laurent-Polz, Institut Nonlinéaire de Nice, 1361 route des Lucioles, 06560 Valbonne, France.

Anna Litvak-Hinenzon, Mathematics Institute, University of Warwick, Coventry CV4 7AL, UK.

Ken Meyer, Department of Mathematical Sciences, University of Cincinnati, PO Box 210025, Cincinnati, OH 45221-0025, USA.

James Montaldi, School of Mathematics, University of Manchester, Manchester, UK.

Tudor Ratiu, Département de Mathématiques, Ecole Polytechnique Fédérale de Lausanne, 1015 Lausanne, Switzerland.

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Esmeralda Sousa Dias, Departamento de Matemática, Instituto Superior Técnico, Av. Rovisco Pais 1049-001, Lisbon, Portugal.

Bob Rink, Department of Mathematics, Imperial College of Science Technology and Medicine, London SW7 2AZ, UK.

Dmitrií Sadovskií, Université du Littoral, MREID, 145 av. Maurice Schumann, 59140 Dunkerque, France.

Luca Sbano, Mathematics Institute, University of Warwick, Coventry CV4 7AL, UK.

Glaucio Terra, Instituto de Matematica e Estatística da Universidade de Sao Paulo, Rua do Matão 1010, Cidade Universitária, CEP 05508-090, São Paulo, SP, Brazil.

Răzvan Tudoran, Departamentul de Matematică, Universitatea de Vest, RO1900 Timișoara, Romania.

Johan Matheus Tuwankotta, Department of Mathematics, Bandung Institute of Technology Jl. Ganesa no. 10, Bandung, 40132 Jawa Barat, Indonesia.

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Preface

In the summers of 2000 and 2001, we organized two European Summer Schools in Geometric Mechanics. They were both held in the wonderful environment provided by the village-cum-international conference centre at Peyresq in the Alpes de Haute Provence in France, about 100km North of Nice. Each school consisted of 6 short lecture courses, as well as numerous short talks given by participants, of whom there were about 40 at each school. The majority of participants were from Europe with a few coming from West of the Atlantic or East of the Urals, and we were pleased to see a number of participants from the first year returning in the second. Several of the courses and short talks led to collaborations between participants and/or lecturers.

The summer schools were funded principally by the European Commission under the High-Level Scientific Conferences section of the Fifth Framework Programme. Additional funding was very kindly provided by the *Fondation Peiresc*. The principal aim of the two schools was to provide young scientists with a quick introduction to the geometry and dynamics involved in geometric mechanics and to bring them to a level of understanding where they could begin work on research problems. The schools were also closely linked to the Mechanics and Symmetry in Europe (MASIE) research training network, organized by Mark Roberts, and several of the participants went on to become successful PhD students or postdocs in MASIE.

Of the lecture courses, seven have been written up for this book—mostly by the participants themselves with varying degrees of collaboration from the lecturers. The book is divided into 6 chapters, each representing a course of 5 or 6 lectures, with the exception of Ratiu's which are taken from two courses. The notes on Stability in Hamiltonian systems by Rink and Tuwankotta based on Meyer's lectures on N -body

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problems have been placed first as they require the least background knowledge. They cover not only Lyapounov's and Dirichlet's stability theorems but also the *instability* theorem of Chetaev, with applications to the restricted 3-body problem. Second are the notes from Ratiu's courses which give an introduction to the mathematical formalism of geometric mechanics, beginning with the Hamiltonian, Lagrangian and Poisson formalisms, and continuing with aspects of reduction and reconstruction, the whole being laced with numerous examples, and including some material on Euler-Poincaré equations. This last topic is the basis of the third set of lecture notes: Holm's course on the Euler-Poincaré approach to fluid dynamics, showing especially how this approach helps to model the multiscale physics involved.

The fourth chapter contains Cushman's lectures on the global geometry of integrable systems, describing particularly the monodromy in such systems, which has recently proved to be so important in explaining some features of molecular spectra. When integrability breaks down, one requires KAM theory which is described in Broer's lectures presented in the following chapter. The theory is described there for dissipative systems, showing how quasiperiodic attractors persist and bifurcate in families of systems, but applies also to conservative systems as is described in the appendix to that chapter.

The final chapter consists of (a slightly expanded version of) Montaldi's lecture course on Hamiltonian bifurcations in symmetric systems. These deal firstly with bifurcations near equilibria including Hamiltonian-Hopf bifurcation, and then with bifurcations of relative equilibria.

We believe all the participants and lecturers would like to join us in thanking Mme. Mady Smets and the staff of the *Peyresq Foyer d'Humanisme* for their warmth, generosity and hospitality, and for the smooth running of the centre without which the Schools would not have had the academic success they did.