Functional Analysis for Probability and Stochastic Processes. An Introduction

This text is designed both for students of probability and stochastic processes and for students of functional analysis. For the reader not familiar with functional analysis a detailed introduction to necessary notions and facts is provided. However, this is not a straight textbook in functional analysis; rather, it presents some chosen parts of functional analysis that help understand ideas from probability and stochastic processes. The subjects range from basic Hilbert and Banach spaces, through weak topologies and Banach algebras, to the theory of semigroups of bounded linear operators. Numerous standard and non-standard examples and exercises make the book suitable for both a textbook for a course and for self-study.

ADAM BOBROWSKI is a Professor of Mathematics at Lublin University of Technology.

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An Introduction

A. BOBROWSKI

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To the most enthusiastic writer ever – my son Radek.

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Preface

This book is an expanded version of lecture notes for the graduate course "An Introduction to Methods of Functional Analysis in Probability and Stochastic Processes" that I gave for students of the University of Houston, Rice University, and a few friends of mine in Fall, 2000 and Spring, 2001. It was quite an experience to teach this course, for its attendees consisted of, on the one hand, a group of students with a good background in functional analysis having limited knowledge of probability and, on the other hand, a group of statisticians without a functional analysis background. Therefore, in presenting the required notions from functional analysis, I had to be complete enough for the latter group while concise enough so that the former would not drop the course from boredom. Similarly, for the probability theory, I needed to start almost from scratch for the former group while presenting the material in a light that would be interesting for the latter group. This was fun. Incidentally, the students adjusted to this challenging situation much better than I.

In preparing these notes for publication, I made an effort to make the presentation self-contained and accessible to a wide circle of readers. I have added a number of exercises and disposed of some. I have also expanded some sections that I did not have time to cover in detail during the course. I believe the book in this form should serve first year graduate, or some advanced undergraduate students, well. It may be used for a two-semester course, or even a one-semester course if some background is taken for granted. It must be made clear, however, that this book is not a textbook in probability. Neither may it be viewed as a textbook in functional analysis. There are simply too many important subjects in these vast theories that are not mentioned here. Instead, the book is intended for those who would like to see some aspects of probability from the perspective of functional analysis. It may also serve as a (slightly long) introduction to such excellent and comprehensive expositions of probability and stochastic processes as Stroock's, Revuz's and Yor's, Kallenberg's or Feller's.

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It should also be said that, despite its substantial probabilistic content, the book is not structured around typical probabilistic problems and methods. On the contrary, the structure is determined by notions that are functional analytic in origin. As it may be seen from the very chapters' titles, while the body is probabilistic, the skeleton is functional analytic.

Most of the material presented in this book is fairly standard, and the book is meant to be a textbook and not a research monograph. Therefore, I made little or no effort to trace the source from which I had learned a particular theorem or argument. I want to stress, however, that I have learned this material from other mathematicians, great and small, in particular by reading their books. The bibliography gives the list of these books, and I hope it is complete. See also the bibliographical notes to each chapter. Some examples, however, especially towards the end of the monograph, fit more into the category of "research".

A word concerning prerequisites: to follow the arguments presented in the book the reader should have a good knowledge of measure theory and some experience in solving ordinary differential equations. Some knowledge of abstract algebra and topology would not hurt either. I sketch the needed material in the introductory Chapter 1. I do not think, though, that the reader should start by reading through this chapter. The experience of going through prerequisites before diving into the book may prove to be like the one of paying a large bill for a meal before even tasting it. Rather, I would suggest browsing through Chapter 1 to become acquainted with basic notation and some important examples, then jumping directly to Chapter 2 and referring back to Chapter 1 when needed.

I would like to thank Dr. M. Papadakis, Dr. C. A. Shaw, A. Renwick and F. J. Foss (both PhDs soon) for their undivided attention during the course, efforts to understand Polish-English, patience in endless discussions about the twentieth century history of mathematics, and valuable impact on the course, including how-to-solve-it-easier ideas. Furthermore, I would like to express my gratitude to the Department of Mathematics at UH for allowing me to teach this course. The final chapters of this book were written while I held a special one-year position at the Institute of Mathematics of the Polish Academy of Sciences, Warsaw, Poland.

A final note: if the reader dislikes this book, he/she should blame F. J. Foss who nearly pushed me to teach this course. If the reader likes it, her/his warmest thanks should be sent to me at both addresses: bobrowscy@op.pl and a.bobrowski@pollub.pl. Seriously, I would like to thank Fritz Foss for his encouragement, for valuable feedback and for editing parts of this book. All the remaining errors are protected by my copyright.