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F. E. A. Johnson

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*University College London*



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To  
C. T. C. Wall  
who first considered this question.

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