

CLASSICAL MECHANICS

Gregory's *Classical Mechanics* is a major new textbook for undergraduates in mathematics and physics. It is a thorough, self-contained and highly readable account of a subject many students find difficult. The author's clear and systematic style promotes a good understanding of the subject: each concept is motivated and illustrated by worked examples, while problem sets provide plenty of practice for understanding and technique. Computer assisted problems, some suitable for projects, are also included. The book is structured to make learning the subject easy; there is a natural progression from core topics to more advanced ones and hard topics are treated with particular care. A theme of the book is the importance of conservation principles. These appear first in vectorial mechanics where they are proved and applied to problem solving. They reappear in analytical mechanics, where they are shown to be related to symmetries of the Lagrangian, culminating in Noether's theorem.

- Suitable for a wide range of undergraduate mechanics courses given in mathematics and physics departments: no prior knowledge of the subject is assumed
- Profusely illustrated and thoroughly class-tested, with a clear direct style that makes the subject easy to understand: all concepts are motivated and illustrated by the many worked examples included
- Good, accurately-set problems, with answers in the book: computer assisted problems and projects are also provided. Model solutions for problems available to teachers from www.cambridge.org/Gregory

The author

Douglas Gregory is Professor of Mathematics at the University of Manchester. He is a researcher of international standing in the field of elasticity, and has held visiting positions at New York University, the University of British Columbia, and the University of Washington. He is highly regarded as a teacher of applied mathematics: this, his first book, is the product of many years of teaching experience.

Cambridge University Press
0521534097 - Classical Mechanics: An Undergraduate Text
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*Bloody instructions, which, being taught,
Return to plague th' inventor.*

SHAKESPEARE, *Macbeth*, act I, sc. 7

Front Cover The photograph on the front cover shows Mimas, one of the many moons of Saturn; the huge crater was formed by an impact. Mimas takes 22 hours 37 minutes to orbit Saturn, the radius of its orbit being 185,500 kilometers. After reading Chapter 7, you will be able to estimate the mass of Saturn from this data!

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AN UNDERGRADUATE TEXT

R. DOUGLAS GREGORY

University of Manchester



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CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo

Cambridge University Press
The Edinburgh Building, Cambridge CB2 2RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9780521826785

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First published 2006

Printed in the United Kingdom at the University Press, Cambridge

A catalogue record for this publication is available from the British Library

ISBN-13 978-0-521-82678-5 hardback
ISBN-10 0-521-82678-0 hardback

ISBN-13 978-0-521-53409-3 paperback
ISBN-10 0 521 53409-7 paperback

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Preface

Information for readers

What is this book about and who is it for?

This is a book on **classical mechanics** for **university undergraduates**. It aims to cover all the material normally taught in classical mechanics courses from Newton's laws to Hamilton's equations. If you are attending such a course, you will be unlucky not to find the course material in this book.

What prerequisites are needed to read this book?

It is expected that the reader will have attended an elementary **calculus** course and an elementary course on **differential equations** (ODEs). A previous course in mechanics is helpful but not essential. *This book is self-contained in the sense that it starts from the beginning and assumes no prior knowledge of mechanics.* However, in a general text such as this, the early material is presented at a brisker pace than in books that are specifically aimed at the beginner.

What is the style of the book?

The book is written in a crisp, no nonsense style; in short, there is no waffle! The object is to get the reader to the important points as quickly and easily as possible, consistent with good understanding.

Are there plenty of examples with full solutions?

Yes there are. Every new concept and technique is reinforced by **fully worked examples**. The author's advice is that the reader should think how he or she would do each worked example *before* reading the solution; much more will be learned this way!

Are there plenty of problems with answers?

Yes there are. At the end of each chapter there is a large collection of problems. For convenience, these are arranged by topic and trickier problems are marked with a star. **Answers are provided to all of the problems.** A feature of the book is the inclusion of computer assisted problems. These are interesting physical problems that cannot be solved analytically, but can be solved easily with computer assistance.

Where can I find more information?

More information about this book can be found on the book's homepage

<http://www.cambridge.org/Gregory>

All feedback from readers is welcomed. Please e-mail your comments, corrections and good ideas by clicking on the comments button on the book's homepage.

Information for lecturers

Scope of the book and prerequisites

This book aims to cover all the material normally taught in undergraduate mechanics courses from Newton's laws to Hamilton's equations. It assumes that the students have attended an elementary calculus course and an elementary course on ODEs, but no more. The book is self contained and, in principle, it is not essential that the students should have studied mechanics before. However, their lives will be made easier if they have!

Inspection copy and Solutions Manual

Any lecturer who is giving an undergraduate course on classical mechanics can request an **inspection copy** of this book. Simply go to the book's homepage

<http://www.cambridge.org/Gregory>

and follow the links.

Lecturers who adopt this book for their course may receive the **Solutions Manual**. This has a **complete set of detailed solutions** to the problems at the end of the chapters. To obtain the Solutions Manual, just send an e-mail giving your name, affiliation, and details of the course to solutions@cambridge.org

Feedback

All feedback from instructors and lecturers is welcomed. Please e-mail your comments via the link on the book's homepage

Acknowledgements

I am very grateful to many friends and colleagues for their helpful comments and suggestions while this book was in preparation. But most of all I thank my wife Win for her unstinting support and encouragement, without which the book could not have been written at all.