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James C. Robinson
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Some of the chapters, and some sections within other chapters, are marked with an asterisk (*). These parts of the book contain material that either is more advanced, or expands on points raised elsewhere in the text.

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