AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS

This refreshing, introductory textbook covers standard techniques for solving ordinary differential equations, as well as introducing students to qualitative methods such as phase-plane analysis. The presentation is concise, informal yet rigorous; it can be used for either one-term or one-semester courses.

Topics such as Euler's method, difference equations, the dynamics of the logistic map and the Lorenz equations, demonstrate the vitality of the subject, and provide pointers to further study. The author also encourages a graphical approach to the equations and their solutions, and to that end the book is profusely illustrated. The MATLAB files used to produce many of the figures are provided in an accompanying website.

Numerous worked examples provide motivation for, and illustration of, key ideas and show how to make the transition from theory to practice. Exercises are also provided to test and extend understanding; full solutions for these are available for teachers.

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To
Mum and Dad,
for all their love, help and support.

Contents

Some of the chapters, and some sections within other chapters, are marked with an asterisk (*). These parts of the book contain material that either is more advanced, or expands on points raised elsewhere in the text.

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Preface

The aim of this book is to deal with all of the elementary methods for obtaining explicit solutions of ordinary differential equations, and then to introduce the ideas of qualitative analysis using phase plane techniques. Simple difference equations are also included, since their methods of solution are similar to those for linear differential equations. As well as being, I hope, an internally consistent choice of material, this selection of topics also has the advantage of preparing a student for a basic course on dynamical systems.

The book arose from my unsuccessful efforts to find a suitable text to recommend when I taught the first year Warwick differential equations course. Although there are a number of well-established and successful textbooks that treat this subject (these are discussed, along with other possibilities for further reading, in the final chapter), they seem either to include a large amount of additional material, or to concentrate only on the more advanced topics. I therefore produced a detailed set of lecture notes, which, with the encouragement of Alan Harvey and David Tranah at Cambridge University Press, eventually became this book. My thanks here to all those students who made helpful suggestions while this book was still at the lecture note stage.

Part I contains an informal discussion of the issues of existence and uniqueness of solutions, and treats the standard classes of first order differential equations that can be solved explicitly, as well as covering exact equations and substitution methods.

The first chapter of Part II shows that two linearly independent solutions are needed in order to solve the general homogeneous problem, and also contains a brief treatment of the Wronskian. The remainder of this section treats equations with constant coefficients, concentrating for the most part on the second order case, with higher order equations discussed briefly at the end.

Second order equations with non-constant coefficients are treated in Part III, which covers reduction of order, the method of variation of constants, and series solutions.

Part IV turns aside from differential equations, motivating the study of difference equations by discussing Euler's method of numerical solution. Constant coefficient linear difference equations are covered, and then there are two chapters devoted to nonlinear difference equations. One of these goes beyond the confines of an introductory course and discusses the dynamics of the logistic map in some detail.

Part V treats coupled systems of two linear differential equations, starting with the substitution method that reduces the problem to a second order differential equation in one variable, the most reliable way to find explicit solutions. The remainder of this portion of the book deals with the matrix approach, showing how a calculation of the eigenvalues and eigenvectors of an appropriate matrix is enough to draw the phase portrait. This is done by changing to a coordinate system in which the equation is put into a standard form, providing an illustration of the Jordan canonical form of a matrix.

Part VI uses the methods from Part V in order to draw the phase plane diagrams for a variety of nonlinear systems, with examples taken from mathematical ecology and simple one-dimensional particle systems, including the pendulum. The book ends with a brief discussion of Dulac's criterion and the Poincaré–Bendixson Theorem, a chapter that investigates the complicated dynamics of the Lorenz Equations, and suggestions for further reading.

In addition to those already mentioned above I would like to thank various people who have contributed to this book. I first learned much of the material here from Tristram Jones-Parry at Westminster School, to whom much belated thanks for all his fine teaching many years ago. I also owe a debt of gratitude to all those who taught the course at Warwick before me, shaping its contents and therefore those of this book; in particular, I had useful guidance from the course notes of Alan Newell and Claude Baesens. I am most grateful to Andrew Stuart, who, in encouraging me to emphasise the links with linear algebra, made me fond of a subject that I still remembered with a shudder from my own undergraduate days. Thanks too to James Macdonald, whose 'Swarm of flies' program for his MMath project on the Lorenz equations was the inspiration behind Figure 37.8.

Over the past two months I have been able to think of little except phase planes and drawing figures in MATLAB: my wife, Tania Styles, has managed to endure my many variations on 'come and see this picture of a washing machine' with a smile. Heartfelt thanks to her for this, and, of course, for everything.

Finally, I would particularly like to thank my Ph.D. student, Oliver Tearne, and my father, John, both of whom read this book extremely carefully and made a number of very helpful comments. For whatever imperfections remain, my apologies to them and to my readers.