

PERSPECTIVES IN FLUID DYNAMICS

A Collective Introduction to Current Research

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## Contents

<i>List of Contributors</i>	<i>page</i> viii
<i>Preface</i>	ix
<b>1 Interfacial Fluid Dynamics</b>	
STEPHEN H. DAVIS	
1 Introduction	1
2 Interfacial regions	4
3 Thin films	8
4 Contact lines	31
5 Singularities, corners and cusps	42
6 Discussion	49
<b>2 Viscous Fingering as an Archetype for Growth Patterns</b>	
YVES COUDER	
1 Introduction	53
2 The basis of Saffman–Taylor viscous fingering instability	58
3 The basis of the instability of other physical systems	65
4 The existence of stable curved fronts	69
5 Fractal structures	84
6 Directional growth	93
7 Conclusions	98
<b>3 Blood Flow in Arteries and Veins</b>	
T. J. PEDLEY	
1 Introduction	105
2 Pulse propagation	108
3 Flow in veins and other collapsible tubes	119
4 Arterial wall shear stress and flow separation	137
5 Conclusion	153

**4 Open Shear Flow Instabilities**

PATRICK HUERRE

1	Introduction	159
2	Open shear flows: amplifiers versus oscillators	161
3	Absolute/convective instabilities in parallel flows	167
4	Global instability analyses	197
5	Absolute/convective and local/global instabilities in nonlinear systems	212
6	Epilogue	224

**5 Turbulence**

JAVIER JIMÉNEZ

1	Introduction	231
2	The small scales and the energy cascade	233
3	Inhomogeneity and anisotropy	245
4	Intermittency	249
5	Free-shear flows	255
6	Wall-bounded flows	265
7	Computing turbulence	276
8	Conclusions	283

**6 Convection in the Environment**

P. F. LINDEN

1	Introduction	289
2	Rayleigh–Bénard convection	294
3	Plumes and thermals	303
4	Double-diffusive convection	321
5	Katabatic flows and gravity currents	335
6	Effects of rotation	337

**7 Reflections on Magnetohydrodynamics**

H. K. MOFFATT

1	Introduction	347
2	Fundamental principles	349
3	The Lorentz force and the equation of motion	354
4	Electromagnetic shaping and stirring	356
5	Dynamo theory	367
6	Relaxation to magnetostatic equilibrium	379
7	Concluding remarks	388

**8 Solidification of Fluids**

M. G. WORSTER

1	Introduction	393
2	Some fundamentals of solidification	394
3	Convective heat transfer	401
4	Binary alloys	407
5	Morphological instability and flow	413

*Contents*

vii

6	Mushy layers	419
7	Solidification and convection in mushy layers	429
8	Concluding remarks	444
<b>9</b>	<b>Geological Fluid Mechanics</b>	
	HERBERT E. HUPPERT	
1	The Earth	447
2	Fluid processes in magma chambers	452
3	The propagation of magma through the crust	466
4	Fluid mechanics and thermodynamics of volcanic eruption columns	477
5	Gravity currents: pyroclastic flows, turbidity currents, lava domes	484
6	Extra topics	495
<b>10</b>	<b>The Dynamic Ocean</b>	
	CHRIS GARRETT	
1	Introduction	507
2	Ocean circulation	509
3	The parameterization of small-scale processes	521
4	Inference	527
5	Measurements	531
6	Processes	537
7	Other problems	551
8	Summary and outlook	553
<b>11</b>	<b>On Global-Scale Atmospheric Circulations</b>	
	MICHAEL E. MCINTYRE	
1	Introduction	557
2	Some fundamentals, including anti-friction	561
3	Wave propagation and gyroscopic pumping	564
4	Wave-induced momentum transport: experiment and theory	566
5	Wave breaking, wave filtering, and critical layers	570
6	The general definition of wave breaking	575
7	The Plumb–McEwan experiment and the refrigeration effect	577
8	Historical note: the Michelson–Morley principle	578
9	Material invariants and stratification surfaces	580
10	Stable stratification and balanced flow	586
11	Oscillations about balance: inertia–gravity wave dynamics	590
12	Balanced oscillations: PV inversion and Rossby wave dynamics	599
13	Rossby waves and anti-friction	606
14	The global-mean circulation of the middle atmosphere	613
15	The response to gyroscopic pumping	618
16	Postlude: the oceans, the troposphere, and climate feedback	619
	<i>Index</i>	625

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## Preface

Research in fluid dynamics has been greatly stimulated over the past few decades, on the one hand by technical advances in analysis and experiment and a vast increase in available computational power, and on the other by the increasing recognition of an ever-expanding range of interdisciplinary fields in which fluid dynamics plays a pivotal role. Thus, for example, not only have there been significant developments in relation to our ability to predict and measure the complex nonlinear interactions within fluid flows that can lead to instability, chaos and turbulence, we have also witnessed the involvement of fluid dynamics and fluid-dynamical principles in a greatly expanded range of fields within both the physical and biological sciences.

At the undergraduate level, fluid dynamics is usually taught in courses in applied mathematics, physics or engineering. Yet at graduate and research level, the fluid dynamicist can become involved in astrophysics, biology, metallurgy, oceanography, meteorology, geophysics and much more, including the traditional branches of the engineering sciences. This has led to a tendency for fluid dynamics to become compartmentalized into specialities with different groups often unaware of advances being made in other areas of the subject. Beyond the undergraduate level, texts in fluid mechanics tend to be similarly specialized, and the graduate student is quickly channelled along a narrow path, which can diverge quite rapidly from paths being pursued by others.

In an attempt to counter this divisive tendency, we have in this book brought together eleven distinguished authors to provide under one cover an introduction to their speciality within fluid dynamics. Though the coverage of fluid-dynamical research is in no way exhaustive, the book does nevertheless include a wide range of topics of current interest. The material is intended to be introductory in that it does not assume any prior knowledge of the sub-fields, but the reader will benefit from having a general foundation in fluid dynamics as normally taught at undergraduate level. Authors were charged with being didactic rather than providing a comprehensive survey of the literature surrounding their subject. References have therefore been kept to a minimum, intended primarily to point the student either to seminal works or to review articles; and just occasionally to where more specialized results can be found. The book could be used as an accompaniment to a graduate-level course in fluid dynamics. More generally, we hope that it will be read and enjoyed by fluid-dynamicists of all levels as a means to learn about and appreciate the great breadth and wealth of our subject.

There is a sense of progression in the book: the first few chapters involve laminar flow, then instabilities and turbulence; these are followed by chapters that introduce other physical processes that act on fluids; and the final few chapters deal with geophysical phenomena, which involve many of the



preceding ideas. However, the chapters are essentially self-contained and can be read in any order, while cross-references are included to help the reader to relate ideas met in different contexts.

Laminar flow is the mainstay of undergraduate texts and courses in fluid dynamics, so one might imagine that there is little new to say on the subject. However, when the boundaries of the fluid domain are deformable, either by being elastic or by forming an interface with another fluid, then many different and complex phenomena can occur. Such flows are described in the first three chapters.

The first chapter, by Stephen Davis, introduces the fundamental ideas of interfacial fluid dynamics on scales at which interfacial forces, such as surface tension, are dominant. This contrasts with traditional studies of surface waves, for example, in which the dominant restoring force (gravity) is internal. Many of the examples he uses to illustrate these ideas originate from modern technological processes, e.g. coating, for which the industrial importance of fluid mechanics is self-evident.

Yves Couder, in Chapter 2, introduces a fundamental instability of a moving fluid–fluid interface, the so-called Saffman–Taylor instability, and considers its nonlinear development. Even at small amplitude, it can have deleterious influence on the coating flows mentioned above and it is a major concern in secondary oil recovery, whereby attempts are made to force more oil from an aging reservoir by injecting a displacing fluid. At large amplitude, the instability causes the interface to adopt complex shapes and is a paradigm for many pattern-forming systems both within fluid mechanics and beyond.

Biology is but one example of a field of research in which fluid dynamics is having a huge impact outside of its traditional applications within the physical and engineering sciences. In Chapter 3, Tim Pedley focuses attention on the flows internal to arteries and veins. The fluid-mechanical novelty comes in part from the interactions between the blood flow and the elastic or collapsible properties of the blood vessels. Additionally the multiply-branched arterial and venous systems give rise to varieties of flow separation and shear enhancement that affect the physiology of the vessels and influence the progress of divers diseases.

Within pure hydrodynamics, modern developments have included a greater understanding of instabilities and further transitions to turbulence. Hydrodynamic instabilities usually result from shear, and take place, therefore, within a background state that is flowing. In consequence, many unstable flows are only convectively unstable – disturbances grow downstream of where they are introduced but do not grow with time at any fixed location. In particular, they do not grow at the point at which they are introduced. Under certain conditions, however, temporal growth at a fixed location does occur. The flow is then said to be absolutely unstable. Absolute instability gives rise to turbulent bursts in pipe flow and to sudden increased mixing at a splitter plate, for example within some fuel-injection systems. In Chapter 4, Patrick Huerre gives an introduction to

the concepts of convective and absolute instability and to the mathematical analysis governing such instabilities in the context of open shear flows.

Hydrodynamic instabilities almost invariably lead to turbulence. Turbulence is a vast area of research extending from investigations into its causes and fundamental nature to, at a very practical level, its influence on mechanical structures and on mixing. In Chapter 5, Javier Jiménez covers the essential ideas of how to characterize turbulence, how to measure it and how to simulate it numerically in order to make quantitative predictions in practical situations.

While the mixing of passive tracers by fluid flows is important in many contexts, much more complex interactions can occur when the quantity being transported itself drives or alters the flow. Such is the case in buoyancy-driven convection, described by Paul Linden in Chapter 6. The paradigm is convection driven by temperature gradients but, equally, convection can be driven by gradients in the concentration of dissolved or suspended impurities. A particular focus of the chapter is convection in confined regions and the development of stable stratification from isolated sources of buoyancy. In addition to many geophysical situations, this phenomenon has significant importance for the containment of reactive chemicals and for the natural ventilation of buildings, for example.

Magnetohydrodynamics (MHD), discussed in Chapter 7 by Keith Moffatt, involves flows that are driven by the Lorentz force resulting from a magnetic field, and the advection (transport) of the field by the flow. In this respect it is similar to buoyancy-driven convection, except that the buoyant agent is a vector rather than a scalar. Additional intrigue results from the fact that the magnetic field itself can be generated by the flow, so the possibility exists for self-sustaining dynamo action, although of course some energy source is required to maintain the flow against viscous dissipation and joule heating. Apart from its intrinsic interest and its application to industrial as well as astrophysical flows, MHD provides some insights into the understanding of turbulence, since the magnetic field is transported in the same way as vorticity.

During solidification, say of water into ice, the boundaries of the fluid domain are altered by phase changes resulting from the transport of heat. In Chapter 8, Grae Worster describes some of the fundamentals of solidification, which combines aspects of interfacial fluid dynamics and of convection, discussed in earlier chapters. In many circumstances the solid–liquid interface can become so convoluted as to form a porous medium in which continued heat and mass transfer, fluid flow and phase change can occur. These processes have significant influence on the fabrication of modern materials (alloys and semi-conductors for example) and on the evolution of many geophysical systems.

Geophysical fluid flows have long been a primary motivation for much fluid-mechanical research and involve many of the different phenomena described in earlier chapters. The most readily and extensively measurable of geophysical flows are those of the atmosphere, which affect our weather, our climate and, of more recent concern, the dispersal of anthropogenic

pollutants. The oceans too, as great reservoirs and conveyors of heat and dissolved minerals and gases, have a major impact on climate. Modelling them provides huge challenges for fluid-mechanical prediction, particularly as measurements of their interior are much more difficult to obtain. But even the Earth beneath our feet is in motion, flowing over aeons in response to internal sources of heat and the periodic loading of the crust by successive ice ages.

Starting from the centre of the Earth, Herbert Huppert in Chapter 9 introduces us to many different flows within and of the Earth, from the rapid flows of molten iron in the outer core, which drive the geodynamo, to the eruptions of lava and volcanic ash, visible to us on the Earth's surface. A major focus of the chapter is a systematic discussion of the passage of magma as it percolates through the mantle, evolves within magma chambers, and erupts through volcanic vents either as lava flows or as vast ash-laden plumes into the atmosphere.

The dynamics of the ocean are introduced by Chris Garrett in Chapter 10. Although the global circulation is perhaps of primary concern, particularly in relation to climate prediction, it is effected and influenced by myriad small-scale processes that cause the mixing of different water masses. Garrett elucidates many of these and describes the challenges involved in making sensible interpretations of ocean measurements.

The mathematical prediction of atmospheric motion and properties is highly developed and relied upon daily in weather forecasts. Yet their global-scale evolution poses many predictive challenges, which are being tackled with increasing urgency as we try to understand and then to modify our own influence on climatic change. In Chapter 11, Michael McIntyre reveals the primary mechanisms giving rise to global circulations of the atmosphere that control some of the dominant characteristics of our climate.

The preparation of this volume would not have been possible without the expert help of Linda Drath who copy edited all the text, Mark Hallworth who prepared many of the figures, and Alison Harrison who gave essential secretarial support at all stages.

This book was the inspiration of George Batchelor, who set out a few years ago to prepare a sequel to his famous textbook *An Introduction to Fluid Dynamics*; a sequel that was in fact hinted at in his preface to that text. He realized that the task was beyond one person, the subject having broadened so much in the intervening thirty years, and invited us to help him edit a book that would serve this purpose. He played a full part in choosing the authors and defining the style of the book. It was also his intention to have written this preface. Sadly, George's health declined rapidly, and he died on 30 March 2000 without seeing the completion of the project. George inspired generations of fluid dynamicists through his founding of the *Journal of Fluid Mechanics* in 1956 and its Editorship for more than 40 years, and through his truly remarkable text. We speak for all the authors in wishing to dedicate this book to his memory.

HKM  
MGW

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