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Flora R. Levin
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1 All Deep Things Are Song*

This world is not conclusion;
A sequel stands beyond,
Invisible, as music,
But positive, as sound.

Emily Dickinson

IT IS NOW ABOVE SEVENTEEN HUNDRED YEARS SINCE BACCHIUS the Elder (as he was then called),[1] a Greek writer on music, opened his *Introduction to the Art of Music* with the seemingly artless question: "What is music?"[2] The answer given by Bacchius to that question

[1] Bacchius *Geron*, like other ancient musicologists – Cleonides, Gaudentius, Alypius, Aristides Quintilianus, *et alii* – is today an unknown and obscure figure, leaving no posterity. Yet, he mattered once, so much so, it seems, as to have had his work on music studied by no less a figure than the Emperor Constantine the Great (285–337 A.D.). A curious epigram attached by one Dionysius (himself unknown), to several manuscripts containing Bacchius' treatise on music makes for this interesting possibility. It states: "Bacchius the Elder compiled the keys, modes, melodies and consonances of the art of music. Writing in agreement with him, Dionysius explains that the almighty emperor Constantine was a learned devotee of the arts. For, being a discoverer and dispenser of all the learned disciplines, it is befitting that he was in no wise a stranger to this one." Everything that can be known of Bacchius is reviewed and assessed by Thomas J. Mathiesen, *Apollo's Lyre*, pp. 583–93.

[2] Bacchius' *Introduction to the Art of Music* (*Eisagōgē Technēs Mousikēs*) was first published in 1623 by Frederic Morellus and, in the same year, by Marin Mersenne.

* Thomas Carlyle

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illustrates wonderfully the ancient Greeks' passion for logic: "It is a conceptual knowledge of melody and all that pertains to melody."[3] There is in Bacchius' answer a dynamic reciprocity: melody provides the conditions for music's existence, while music at the same time subsists by virtue of melody. After detailing all the elements that pertain to melody – pitch, interval, consonance, dissonance, scales, modes, modulation, keys, rhythm, and much more – Bacchius arrived at not one but two different definitions of melody. The first is deliberately circular: "It is the fall and rise generated by melodious notes."[4] Such a definition is tantamount to asserting that something is a melody because its constituents are melodious. If Bacchius' definition is in fact tautological, it seems to be so consciously, in order that it be correct on purely musical grounds. It arises from Bacchius' belief that nothing in the world outside of melody can be invoked or enlisted to define anything that

The treatise is written in a question and answer format like that of the Ps.-Aristotle *Problems* (Book 11 *On the Voice* and Book 19 *On Harmonia*) and *The Pythagorean Doctrine of the Elements of Music* by the remarkable Ptolemaïs of Cyrene, of whose work only fragments remain. See Chapters 6 and 7. Bacchius' treatise belongs to the school of Aristoxenus, "The Musician," as he was known in antiquity, who, unlike Bacchius, did leave a considerable posterity. The Greek text from which these translations are drawn is that of K. von Jan, *Musici Scriptores Graeci*, pp. 292–316 (hereafter referred to as Jan). On its English translation, see Chapter 2, note 2. Another has been promised by Jon Solomon, one of the few scholars who has examined the work, in his "*EKBOLE* and *EKLUSIS* in the Musical Treatise of Bacchius," *Symbolae Osloenses* 55 (1980), 122, n. 1. The question that opens the treatise appears at I . 1 (Jan, 292. 3).

3 I. 1 (Jan, 292. 3–4). There is an unmistakably Platonic cast to Bacchius' definition of music. His term *eidēsis* (which I have rendered as "conceptual knowledge") links "knowing" (*eidōs*) with "seeing" (*eidon*) and, as such, has a strong relationship with Plato's *eidōs-eidē*, the terms canonized by Plato as "Form" and "Forms" and abstracted by him into "Ideas." In Plato, the *eidē* are the cause of true knowledge (*epistēmē*), inasmuch as they are the eternal realities that transcend the merely sensible phenomena (*aisthēta*). The *eidē* are therefore the condition of all true philosophical discourses. See *Phaedo* 65D–E; *Parmenides* 135B–C). Bacchius' definition thus recognizes the mental activity (*eidēsis*) of the knowing mind by which the melodic concept of Form (*eidōs*) is grasped.

4 I. 19 (Jan, 297. 22–23). The words for "fall" and "rise" are in Aristoxenian terms *anesis* and *epitasis*, which have the literal meanings, "resolution" and "tension," respectively. These words are discussed below, n. 41.

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lies within the precincts of melody. Bacchius' approach to the nature of melody adumbrates almost eerily what Ludwig Wittgenstein was to observe centuries later:[5] "Die Melodie ist eine Art Tautologie, sie ist in sich selbst abgeschlossen; sie befriedigt sich selbst. [Melody is a form of tautology, it is complete in itself; it satisfies itself.]"

Like Bacchius, Wittgenstein evidently contemplated music not in any sense as a language, but as an activity of some sort. And Bacchius, like Wittgenstein, seems to have treated music as an activity whose subject matter is not in any sense factual, but whose processes and results are somehow equivalent. Accordingly, where Bacchius found logic in melody, Wittgenstein found melody in logic:[6] "Die musicalischen Themen sind gewissen Sinne Sätze. Die Kenntnis des Wesen der Logik wird deshalb zur Kenntnis des Wesens der Musik führen. [Musical themes are in a certain sense propositions. Knowledge of the nature of logic will for this reason lead to knowledge of the nature of music.]"

In Wittgenstein's view, a melody, like a logical proposition, must make sense strictly according to its own terms in order to attain to the condition of music. The implication is that the condition of music, in order to be true to itself, must inhere in the logical constants of melody.

Like a mathematical proposition that uses numerical signs and symbols to designate an intricate truth, a melody employs certain logical constants, which are preserved by notational signs and symbols, in order to make musical sense. The proposition that $7 + 5 = 12$ is necessarily true for a mathematician, whether the symbols stand for apples, peaches, trees, or anything else; it holds true because it defines an unchanging relation, one that holds independently of the objects involved or of the mind that contemplates them. Thus, to ascertain that the mathematical proposition $7 + 5 = 12$ is correct, we do not have to study the universe; we have merely to check the meaning of the numerical symbols. They assert that $7 + 5$ has the same meaning as 12, and this amounts to a mathematical truth – a truth that, by its very nature, is a tautology. So, too, the pitches C–C^1, for example, define a melodic constant, one that stands for nothing in the world save itself. This holds true whether these pitches be struck on a piano, bowed on a violin string, or blown

[5] *Notebooks*, 1914–16, p. 40, 4.3.15.
[6] *Notebooks*, 1914–16, p. 40, 7.2.15.

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on an ancient Greek *aulos*. The melodic constant defined by the pitches C–C^1 is an unchanging relation – that of an octave. C–C^1 means "octave" to a musician and, as such, is a form of tautology.[7]

In the case of mathematics, we perceive the unchanging relation in an arithmetic proposition such as that given earlier, not in virtue of the symbols as phenomenal entities in themselves but by their intervention as symbols standing *for* phenomenal entities; in the case of music, we perceive the unchanging relation not by the intervention of the symbols representing something else but as phenomenal entities in themselves.[8] In other words, the octave C–C^1, whether written in alphabetic notation (as here) or as notes on a staff, *is* the octave C–C^1 at that specific pitch range. Numbers, unlike the musical notes that make up a melody, are needed not only to compose mathematical propositions, but also, if they are to have a specific meaning, to stand for or apply to common objects in the world. It is only when numbers are abstracted from the objects of the world or understood independently of the world that they begin to assume the characteristics of musical notes. Like a musical note, the number 2, say, is not the same as or identical with anything in the world. It *is* 2 and, as such, is not identical with a duo of musicians; rather, the duo of musicians is an instance of the number, 2; and the number, 2, is an instance of itself. The note C, is in this sense an instance of itself.[9] Given such an instance, the subject matter of music, like that of pure

[7] The convention for indicating pitch adopted here is that of Carl E. Seashore, *Psychology of Music*, p. 73. This tautological type of formulation is seen by many as a logical trick that is a favorite device of mathematicians. For example, Susan Langer, *Philosophy in a New Key*, p. 237: "Musical form, they reply, is its own content; it means itself. This evasion was suggested by [Eduard] Hanslick when he said, 'The theme of a musical composition is its essential content.'"

[8] Thus Langer, *op. cit.*, p. 19: "Mathematical constructions are only symbols; they have meanings in terms of relationships, not of substance; something in reality answers to them, but they are not supposed to be items in that reality. To the true mathematician, numbers do not 'inhere in' denumerable things, nor do circular objects 'contain' degrees. Numbers and degrees and all their ilk only *mean* the real properties of real objects."

[9] This is another way of saying that music is non-representational. Because it exhibits its own pure form as its own essence, it is altogether untranslatable into any other medium save itself. It is in this sense wholly tautological, as Wittgenstein observed (see note 5). Cf. Peter Kivy, *Music Alone*, pp. 66–67.

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mathematics, belongs to a realm of idealized abstractions in which the composer of music or the mathematician performs specific operations with an extrawordly creative freedom.

It was perhaps his intuition of this curious affiliation between music and mathematics that led Pythagoras of Samos (sixth century B.C.), the most influential mathematician of antiquity, to make music a matter for serious philosophical reflection. As reported by Athenaeus of Naucratis (160–230 A.D.), an authority on the musical lore of antiquity:[10] "Pythagoras, who occupied so very great a position in philosophy, stands out among the many for having taken up music not as an avocation; indeed, he explains the very being of the whole universe as bound together by music." Pythagoras' intuition along these lines was to produce one of the most momentous discoveries of all time: musical sound is ruled by number. This meant nothing less than that the whole universe, as being bound together by music, must itself be ruled by number.[11]

[10] Athenaeus of Naucratis in Egypt wrote a monumental work entitled *Deipnosophistai* (Sophists at Dinner) sometime after the death of the Emperor Commodus (180–92 A.D.). The work consisted originally of fifteen volumes. Much is lost, but what remains of Books IV and XIV in particular is valuable for preserving information on music and musical practices from much earlier sources such as Pindar (c. 522–c. 446 B.C.), Bacchylides (c. 520–c. 450 B.C.), Damon, the teacher of Socrates (5th century B.C.), Hesiod (c. 700 B.C.), Aristotle (384–322 B.C.), Aristoxenus of Tarentum (b.c. 375 B.C.; his date of death is unknown), the leading musician and musical theorist of antiquity. The passage quoted here is from Book XIV, 632b.

[11] The literature on Pythagoras and his discovery of the mathematical ratios underlying the production of musical pitches is vast enough to make up an entire library. Some of the comprehensive studies are: P.-H. Michel, *De Pythagore à Euclide*; P. Kucharski, *Étude sur la doctrine pythagoricienne de la tétrade*; E. Frank, *Plato und die sogenannten Pythagoreer*; A. E. Chaignet, *Pythagore et la philosophie pythagoricienne*; A. Delatte, *Études sur la littérature pythagoricienne*; H. Thesleff, *An Introduction to the Pythagorean Writings of the Hellenistic Period*; H. Thesleff, *The Pythagorean Texts of the Hellenistic Period*; W. Burkert, *Lore and Science in Ancient Pythagoreanism*; W. K. C. Guthrie, *A History of Greek Philosophy*, Vol. 1: *The Earlier Presocratics and the Pythagoreans*; J. A. Philip, *Pythagoras and Early Pythagoreanism*; K. S. Guthrie, *The Pythagorean Sourcebook and Library*; J. Godwin, *The Harmony of the Spheres: A Sourcebook of the Pythagorean Tradition*. Pythagoras' reputation is summed up accordingly by G. E. Owen, *The Universe of the Mind*,

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On the basis of the truths arrived at by mathematical means, Pythagoras and his followers could think of music's elements as concrete realities linked by number to nature's own divine proportions. At the same time, they could think of music itself as the expression in sound of those same proportions by which nature asserts her divine symmetry. This being the case, the universal order of things could be said to have its counterpart in the underlying structures of harmonic theory. The notion that music owes its life to mathematics, and that the universe, by the same agency, owes its soul to *harmonia* – the attunement of opposites – took hold of human imagination from its first utterance and has transfixed it for the millennia.[12]

It was during that brilliantly fecund period when Aeschylus was producing his dramas and Pindar his *Odes* that Pythagoras made the discovery about music, a discovery that reverberates to this day. As far as we can ascertain, no Greek-speaking person had ever committed the story of the discovery to writing before Nicomachus of Gerasa, the "Pythagorean," so-called, a once famous mathematician living at the turn of the century, 100 A.D.[13] According to Nicomachus, Pythagoras had long been pondering the problem of how to translate the musical sounds he produced on the strings of his lyre into some sort of concrete form that his eye could see and his mind could contemplate. While deliberating about this problem, he happened to be walking by a smithy

p. 26: "Born about 573 B.C.E. on Samos, according to legend Pythagoras became the ideal of mathematics, a philosopher and a prophet, apotheosized in his own lifetime by his own society."

12 The moment Pythagoras discovered that the lengths of a vibrating string sounding a fundamental pitch, its fifth, its fourth, and its octave, are in the ratios 2: 3: 4, he heard the *harmonia* of the universe and defined the ordering of its elements (*stoicheia*) in numerical terms. From now on, nature and all its properties were to be found in the science of number. Cf. H. E. Huntley, *The Divine Proportion*, pp. 51–56. It is thus to Pythagoras that we owe the first conception of the universe as a harmony patterned on music. For because sounds were shown to be the embodiment of numbers, it was conceivable that *mundum regunt numeri*, that the world was in fact ruled by number.

13 Nicomachus tells the story of Pythagoras' discovery in his *Harmonikon Enchiridion* (*Manual of Harmonics*), Ch. 6 (Jan, 245.19–248). *The Manual*, together with ten *Excerpta* from a lost work of Nicomachus, is translated by Levin, *The Manual of Harmonics of Nicomachus the Pythagorean*.

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where, by sheer good fortune, he heard the smith's hammers beating out on the anvil a whole medley of pitches. These registered on his ear as the same consonances that he could produce on his lyre-strings – the octave, the fifth, and the fourth – as well as the dissonance separating the fourth from the fifth – the whole-tone.

Intuiting that the size of the sounding body had something to do with the differences between the pitches he heard, Pythagoras ran into the smithy, conducted a series of experiments, which he repeated at home, and came upon the elegantly simple truth about musical sound: the pitch of a musical sound from a plucked string depends upon the length of the string. This led him to discover that the octave, the fifth and the fourth, as well as the whole-tone, are to each other as the ratios of the whole numbers. These, the harmonic ratios, as they came to be called, are all comprehended in a single construct: 6:8 :: 9:12. This means that the octave may be represented by 12:6 or 2:1; the fifth by 12:8 or 3:2; the fourth by 12:9 or 4:3. Moreover, the fifth may also be represented by 9:6 = 3:2, and the fourth by 8:6 = 4:3. The whole-tone, being the difference between the fifth and the fourth, is represented by 9:8, a ratio that cannot on division yield a pair of whole numbers. That is, dividing 9:8 gives $3:2\sqrt{2}$, an irrational or *alogos* number.

In the deceptively simple construct – 6:8 :: 9:12 – there are contained all the primary constituents of music's elemental structure and, inferentially, the harmonic symmetry of the universe. What is more, there is embodied in this set of ratios the original Pythagorean *tetraktys*, the ensemble of the four primary numbers – 1, 2, 3, 4 – that became the corner-stone of the Pythagorean philosophy of number. From the intrinsic properties of this – the *tetraktys* of the *decad* – the sum of whose terms equals the number 10, there were harvested in turn the theory of irrationals, the theory of means and proportions, the study of incommensurables, cosmology, astronomy, and the science of acoustics.[14] Because

[14] The original Pythagorean *tetraktys* or quaternary represented the number 10 in the shape of a perfect triangle composed of four points on each side. It showed at a glance how the numbers 1, 2, 3, 4 add up to ten. The number 10 was thus regarded as sacred, since it comprehended the equivalent in each of its terms to the universal components of the point (= 1), the line (= 2), the triangle (= 3), the pyramid (= 4). What is more, it contained in its terms the basic elements

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"truth is truth to the end of reckoning," the harmonic properties of these numbers are as true today as they were for Pythagoras when he first discovered them. But a by-product of Pythagoras' discovery turns out to be the most stubborn problem in the science of acoustics: the incommensurability of the whole-tone. Interestingly enough, the same irrational number that appears on the division of the whole-tone was found by Pythagoras to obtain between the side of a square and its diagonal. In the one case, that of the whole-tone in the ratio 9:8, the division for obtaining a semi-tone, or one half of a whole-tone, produces the square root of 2. In the case of the geometric square, the Pythagorean theorem demonstrates that if the length of the side is 1 inch, the number of inches in the diagonal is also the square root of 2. As George Owen explains, this discovery was greeted with little joy by the Pythagoreans:[15]

> Having founded their order on the purity of number, the Pythagoreans were dismayed to discover the existence of the irrational number. Such numbers may well have played an important role in their mystery rites. Soon after the death of Pythagoras, Hippasus {an acoustical expert from Metapontum} ... communicated his [Pythagoras'] views and some of the Pythagorean doctrines to outsiders. For this he and his followers were expelled from the society.

The problem of irrationality for mathematicians was eventually resolved by a feat of intellectual genius: the reduction of arithmetic to logic.[16]

of music: 2:1 (octave), 3:2 (fifth), 4:3 (fourth), 4:1 (double octave). Cf. Owen (above, n. 11), pp. 27–28. The literature on the *tetraktys* is vast. For pertinent references, see F. R. Levin, *The Harmonics of Nicomachus and the Pythagorean Tradition*, p. 65, n. 62.

15 Owen (note 11), p. 32. Just as no fraction will express exactly the length of the diagonal of a square, so too, no fraction will express exactly the size of the musical interval that registers on the ear as a semi-tone. The division of both the diagonal and the whole-tone yield the same irrational. As Bertrand Russell, *Introduction to Mathematical Philosophy*, p. 67 observed: "This seems like a challenge thrown out by nature to arithmetic. However the mathematician may boast (as Pythagoras did) about the power of numbers, nature seems able to baffle him by exhibiting lengths which no numbers can estimate in terms of the unit."

16 The fundamental thesis to which Bertrand Russell devoted his *Principles of Mathematics* (1903) is that all the constants that occur in pure mathematics are

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For musicians, the incommensurability of the whole-tone was resolved by an intellectual feat of no less brilliance: the well-tempered system of tuning.[17]

Because of the deep and far-reaching implications that Pythagoras' discovery had for such fundamental branches of knowledge as mathematics, cosmology, and astronomy – implications that extended far beyond its immediate utility in converting the sensory distinctions of pitch and interval into objective numerical form – it was treated by the ancients as a divine revelation. The story of Pythagoras' experiments with stretched strings and various other instruments such as panpipes, monochords, auloi, and triangular harps, and the dazzling discovery to which these experiments led, having once been told by Nicomachus, was passed along through the centuries in an unbroken tradition from one (now) obscure writer to another: from the musical theorist Gaudentius, surnamed "The Philosopher" (2nd or 3rd century A.D.), to the Neo-Platonist, Iamblichus (c. 250–c. 325 A.D.), to the biographer of ancient philosophers, Diogenes Laertius (3rd century A.D.), to the Roman grammarian, Censorinus (3rd century A.D.), thence to

logical constants; that, hence, the truths of mathematics can be derived from logical truths. As Russell argued, in order to deal with the two great sources of irrational numbers – the diagonal of the square and the circumference of the circle – the logical notion of spatial continuity had to have been introduced as an axiom *ad hoc* (pp. 438–39). In thus generalizing the notion of spatial continuity to the utmost, Russell created a set of new deductive systems, in which traditional arithmetic – which had laid bare the irrationality of the diagonal of a square and the circumference of the circle – was at once dissolved and enlarged. He observes in *Mathematics and Logic*, p. 196: "... we have, in effect, created a set of new deductive systems, in which traditional arithmetic is at once dissolved and enlarged ..."

17 It will be argued below (pp. 202ff.) that Aristoxenus, the major source of such logically-minded musical theorists as Bacchius the Elder and others, attacked the problem of the irrationality of musical space in much the same way as that described by Russell; that is, he began by introducing the logical notion of spatial continuity as a necessary axiom *ad hoc*. This led him to dissolve and enlarge what had been for him traditional mathematics – Pythagorean harmonics. Logic and the establishment of logical constants led him finally to the monumental accomplishment for which he is credited in these pages: equal temperament.

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Chalcidius, honored for his commentary on Plato's *Timaeus* (4th century A.D.), to the illustrious polymath, Macrobius, known for his commentary on Cicero's *Somnium Scipionis* (4th–5th century A.D.), to Fulgentius (c. 467–532 A.D.), famed for his writings on mythology. Finally, the tale of Pythagoras' discovery was deposited with the towering scholar, Boethius (480–524 A.D.), who honored Nicomachus by translating his account into Latin. Thereafter, it was preserved for posterity by one of the most important links between the scholarship of antiquity and that of the Middle Ages, Isidore of Seville (570–636 A.D.),[18] reappearing with various embellishments in the account of the Patriarch, André de Crète, surnamed Hagiopolites (died 8th century A.D.).[19]

Pythagoras' revelation of the affinity between music, mathematics, and philosophy has lost none of its majesty in these many retellings; if anything, it has gained power and importance through the numerous scientific investigations it set into motion long after its institution in the scientific literature.[20] Indeed, according to general opinion, no

[18] Another tradition, existing quite apart from the Pythagorean, connects the legendary discovery of the concordant intervals produced on the anvil with the smith's hammers to the Idean Dactyls, so-called, the dwarfish craftsmen of ancient Phrygian chronicles – ancient *Nibelungen*, as it were – the servants of the Asian Goddess, Rhea Kybele. This tradition is discussed by Eric Werner, *The Sacred Bridge*, pp. 376–77. Still another tradition is to be found in *Genesis* 4.21, where the discovery of music and the harp is attributed to Jubal, the descendant of Cain. The inference is that the connection between music and number had been arrived at independently of Pythagoras and quite possibly long before him. This is argued by Otto Neugebauer, *The Exact Sciences in Antiquity*, pp. 35–36.

[19] The account of the Hagiopolites differs interestingly from that of Nicomachus. For whereas Nicomachus portrays Pythagoras as experimenting with various kinds of instruments, finally settling on the monochord as the most convenient for his purposes, the Hagiopolites describes Pythagoras as building a four-stringed lute-type instrument, an instrument which he named *Mousikē*. This account is to be found in one place only, that of A. J. H. Vincent, in the sixteenth volume of a very rare and beautiful book entitled *Notice sur divers manuscrits grec relatif à la musique*, pp. 266–68.

[20] Thus, Sir Thomas Heath. *Aristarchus*, pp. 46–47: "The epoch-making discovery that musical tones depend on numerical proportions, the octave representing the proportion of 2:1, the fifth 3:2, and the fourth 4:3, may with sufficient certainty be attributed to Pythagoras himself, as may the first exposition of

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