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978-0-521-51600-6 - Dark Energy: Theory and Observations
Luca Amendola and Shinji Tsujikawa
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DARK ENERGY

Theory and Observations

Dark energy, the mysterious cause of the accelerating expansion of the Universe, is one of the most important fields of research in astrophysics and cosmology today. Introducing the theoretical ideas, observational methods and results, this textbook is ideally suited to graduate courses on dark energy, and will also supplement advanced cosmology courses.

Providing a thorough introduction to this exciting field, the textbook covers the cosmological constant, quintessence, k-essence, perfect fluid models, extra-dimensional models, and modified gravity. Observational research is reviewed, from the Cosmic Microwave Background to baryon acoustic oscillations, weak lensing, and cluster abundances. Every chapter ends with problems, with full solutions provided, and any calculations are worked through step-by-step.

LUCA AMENDOLA is Professor of Physics at the University of Heidelberg, Germany. He is also an astronomer at the Italian National Institute for Astrophysics, Rome, and has authored more than 100 papers in international journals and conference proceedings.

SHINJI TSUJIKAWA is Associate Professor at Tokyo University of Science in Japan, where he teaches cosmology and relativity. He has published more than 80 papers in international refereed journals and has over 60 collaborators worldwide.

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Theory and Observations

LUCA AMENDOLA
Institut für Theoretische Physik, Universität Heidelberg/
Osservatorio Astronomico di Roma

SHINJI TSUJIKAWA
Department of Physics, Faculty of Science,
Tokyo University of Science

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Preface

Perhaps the first recognition that the matter composing the universe may be different from the one we touch and experience every day has been put in writing by early Greek philosophers and by Aristotle in particular. In his work *On the Heavens*, Aristotle argues that the nature and movement of the stars and planets is so fundamentally different from Earth-like elements that a new substance is required, a "*bodily substance other than the formations we know, prior to them all and more divine than they*."[1] Later on this cosmic element came to be called *quintessentia*, or fifth element, and drawing on Plato's classification of the elements a dodecahedron's figure was associated with it.

More than two thousand years later, astrophysicists have begun to pile up evidence that a new form of matter pervades our Universe. This idea is based on observations that reminds one of Aristotle's thoughts: the global movement we observe in distant reaches of our cosmos is unexplainable by ordinary matter. All the matter we see on Earth, in the solar system, inside our Galaxy or in similar structures across the Universe has a small or negligible positive pressure and clumps under the influence of gravity. An expanding Universe filled with this form of matter would by necessity slow down. But in 1998, astronomers studying the global expansion by the use of supernovae found that their observed luminosities can be explained only by an accelerated expansion of the Universe. After a full decade of more observations, more analyses and more interpretations, we still cannot find a better explanation than invoking something new, a new force or a new matter, that acts "*on the heavens*." This new form of matter, called *dark energy*, is what this book is about.

We do not know the nature of dark energy yet. We are beginning to characterize its properties in several ways, from its abundance to its dynamics, but we know still very little about it. The simplest explanation, an energy associated with the

[1] Aristole, *On the Heavens*, I, 2.

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vacuum, was already proposed on a totally different basis by Einstein under the name of universal (Einstein's original term) or cosmological constant[2] Λ. But the freedom still allowed by observations has unleashed theorists' imagination and many interpretations of dark energy have been advanced. Dark energy is indeed a general label for what we do not know about the large-scale properties of our Universe, its history and its geography.

So why a book on dark energy? One of the immediate consequences of the discovery of the cosmic acceleration and the hypothesis of dark energy has been that astrophysicists and particle physicists, both theorists and experimentalists, have been drawn together into this new field of research, with their own languages and methods. We believe that this has created the need for a resource that allows scholars and students to apprehend the basis of dark energy research in an interdisciplinary way. This book introduces the main theoretical ideas on dark energy and at the same time the basics of the observational methods and results. There are several reviews that cover parts of the dark energy research but not a book that could be used as a starting point to advanced and more topical material.

This book can be used as a companion text for an advanced cosmology course, covering several areas that complement modern cosmology textbooks or as a stand-alone text for graduate or post-graduate courses on dark energy. It is also addressed to newcomers in the field that wish to identify the main lines of the current research. Finally, we have in mind also researchers in the dark energy field who need to explore other sides of the discipline and would like to have a handy reference for many results and topics scattered in the literature. For most of the book we assume knowledge of General Relativity and basic cosmology at the graduate level and little more. Some more advanced sections (especially Sections 6.5 and 7.4) require also a background in quantum field theory but they can be left aside without prejudice. Whenever possible we give a fairly comprehensive review of the tools required for further material, for instance we introduce the concepts of statistics and of cosmological perturbation theory that are needed for understanding the subsequent chapters. We provide 44 fully solved problems with some detailed calculations, which will help the reader to test his/her understanding.

The immense impact on cosmology of the dark energy concept is witnessed by the many projects around the world aiming at collecting more and more data, from large-scale galaxy surveys to weak lensing surveys, from cosmic microwave observations to gamma-ray bursts. The very nature of the issue at stake, the study of a component that determines to a large extent the present and future cosmic dynamics, has generated a great diversity of theoretical and observational approaches. One

[2] As a curious coincidence, Aristotle first proposed his eternal and incorruptible "cosmic substance" in the book Λ (i.e. the twelfth book) of the *Metaphysics*. "The Lambda Book," as it was called in the middle ages, was singled out as the highest point of Aristotle's metaphysics.

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can attack the problem from the point of view of an exotic matter component, or of a non-Einsteinian gravity force, or invoking multidimensional effects. Similarly, one can employ a very diverse array of observations, from standard candles to standard clocks, from supernovae to quasar multiple images. Many of the hypotheses and methods will have a strong impact on cosmology even beyond the dark energy problem. This book tries to extract from this variety the core teachings: methods, suggestions, hypotheses, and techniques that are shaping our knowledge of the cosmos. Many of these, we reckon, will remain with us for many years.

Although the diversity of approaches is one of the hallmarks of dark energy research, we could not possibly cover all the ideas discussed so far. Up to 2009 the number of papers that include the words "dark energy" or "cosmological constant" in the title has been over 3700. We tried to discuss all the driving ideas but not all possible implementations. In doing so we certainly missed some interesting contributions; we apologize in advance to our colleagues.

We thank all our collaborators on the topics in this book, namely, Carlo Baccigalupi, Amedeo Balbi, Marco Baldi, Kazuharu Bamba, Riccardo Barbieri, Bruce A. Bassett, Silvio Bonometto, Stefano Borgani, Robert Brandenberger, Carlo Burigana, Paolo Cabella, Gianluca Calcagni, Gabriela C. Campos, Salvatore Capozziello, Daniela Carturan, Christos Charmousis, Edmund J. Copeland, Pier Stefano Corasaniti, Stephen C. Davis, Antonio De Felice, Cinzia Di Porto, Stephane Fay, Fabio Finelli, Radouane Gannouji, Mohammad Reza Garousi, Maurizio Gasperini, Chao-Qiang Geng, Emanuele Giallongo, Fabio Giovi, Burin Gumjudpai, Zong-Kuan Guo, Soo A. Kim, Martin Kunz, Maxim Libanov, Andew R. Liddle, Roy Maartens, Andrea Macciò, Kei-ichi Maeda, Roberto Mainini, Elisabetta Majerotto, Martin Makler, Matteo Martinelli, Alessandro Melchiorri, Shuntaro Mizuno, Bruno Moraes, David F. Mota, Tapan Naskar, Savvas Nesseris, Shin'ichi Nojiri, Sergei Odintsov, Junko Ohashi, Nobuyoshi Ohta, Sudhakar Panda, Eleftherios Papantonopoulos, David Parkinson, Alessandro Pasqui, Valeria Pettorino, Yun-Song Piao, Federico Piazza, David Polarski, Miguel Quartin, Claudia Quercellini, R. R. R. Reis, Rogerio Rosenfeld, Valery Rubakov, M. Sami, Domenico Sapone, Parampreet Singh, Alexei Starobinsky, Takashi Tamaki, Takayuki Tatekawa, Reza Tavakol, Domenico Tocchini-Valentini, Alexey Toporensky, Peter V. Tretjakov, Roberto Trotta, Kotub Uddin, Carlo Ungarelli, Ioav Waga, David Wands, John Ward, Christof Wetterich, Jun'ichi Yokoyama, and Xinmin Zhang.

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L.A. and S.T.

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I wish to honor the memory of Franco Occhionero, long-time friend and teacher: you opened to me the magic lamp of cosmology. I also thank the staff and colleagues at the Osservatorio Astronomico di Roma and the Director Emanuele Giallongo, for support, patience, collaboration, help, friendship, and everything that makes work possible, pleasant and productive. Thanks to the University of Heidelberg for the new exciting opportunities they are offering to me. Special thanks to Silvio Bonometto, Roberto Buonanno, Roberto Scaramella and Christof Wetterich for collaboration, advice, support, and friendship. Special thanks to Claudia Quercellini, Ioav Waga, Miguel Quartin, Fabio Finelli, Cinzia di Porto for comments on the draft and for the long-term friendship and collaboration. Thanks also to Adam Amara, Enzo Branchini, Francisco Castander, Gigi Guzzo, Tom Kitching, Rocky Kolb, Martin Kunz, Roberto Maoli, Anais Rassat, Alexandre Refregier, Yun Wang, Jochen Weller, and all the other DUNE/Euclid and NASA/SWG collaborators, too many to be listed here, from whom I learned a lot in terms both of science and of scientific collaboration.

L.A.

I am grateful to Kei-ichi Maeda for giving me a chance to work as a cosmologist, in spite of the fact that I was majoring mathematics before entering a master course of physics in 1996. This was a correct decision because I witnessed enormous progress of observational and theoretical cosmology from the late 1990s, especially in the field of dark energy. I am also thankful to the members of Tokyo University of Science and Gunma National College of Technology, especially to Antonio De Felice, Hitoshi Fujiwara and Junko Ohashi, for their kind support and help. I also thank Bruce A. Bassett and Roy Maartens, who kindly helped me in many aspects during my stay in Portsmouth in 2003. I am grateful to M. Sami for a long-term collaboration about dark energy.

S.T.

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Frequently used symbols

Symbol	Definition
G	Gravitational constant ($G = 6.67 \times 10^{-8}\,\mathrm{cm^3\,g^{-1}\,sec^{-2}}$)
$m_{\rm pl}$	Planck mass ($m_{\rm pl} = 1/\sqrt{G} = 1.2211 \times 10^{19}$ GeV)
$M_{\rm pl}$	Reduced Planck mass ($M_{\rm pl} = 1/\sqrt{8\pi G} = 2.4357 \times 10^{18}$ GeV)
κ	$\sqrt{8\pi G}$
a	Scale factor of the Universe (with the present value $a_0 = 1$)
t	Cosmic time
η	Conformal time: $\eta = \int a^{-1}\,\mathrm{d}t$
N	Number of e-foldings: $N = \ln a$
$\dot{}$	Derivative with respect to t
$'$	Derivative with respect to η (or $N = \ln a$ in Chapters 11 and 12)
z	Redshift: $z = a_0/a - 1$
d_A, d_L	Angular diameter distance, luminosity distance
$H, \mathcal{H}$	Hubble parameter: $H = \dot{a}/a$, conformal Hubble parameter $\mathcal{H} = aH$
$H_0,\ h$	Present Hubble parameter: $H_0 = 100\,h$ km sec^{-1} Mpc^{-1}
$E(z)$	Hubble parameter normalized by H_0: $E(z) = H(z)/H_0$
ρ	(Energy) Density
P	Pressure
w	Equation of state: $w = P/\rho$
$w_{\rm eff}$	Effective or total equation of state: $w_{\rm eff} = -1 - 2\dot{H}/(3H^2)$
K	Curvature of the Universe
R	Ricci scalar
$\Omega^{(0)}$	Density parameter at the present epoch ($z = 0$)
c_s	Sound speed
r_s	Sound horizon: $r_s(\eta) = \int_0^\eta \mathrm{d}\tilde{\eta}\, c_s(\tilde{\eta})$
$\Phi,\ \Psi$	Gravitational potentials
T	Temperature

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Symbol	Definition
Θ	Temperature perturbations: $\Theta = \delta T/T$
k	Comoving wavenumber
$P(k)$	Power spectrum of perturbations
ℓ	Spherical harmonic multipoles
C_ℓ	Multipole power spectrum
$\mathcal{R}$	CMB shift parameter
δ	Density contrast
D	Growth function
b	Bias (ratio of galaxy to total matter perturbations)
Λ	Cosmological constant
S	Action
$g_{\mu\nu}$	Metric
$G_{\mu\nu}$	Einstein tensor
$T_{\mu\nu}$	Energy-momentum tensor
ϕ	Scalar field
$V(\phi)$	Scalar-field potential in the Einstein frame
$U(\phi)$	Scalar-field potential in the Jordan frame
$\mathcal{L}$	Lagrangian density (also log-likelihood)
λ	Slope of the potential defined by $\lambda = -V_{,\phi}/(\kappa V)$
$\hat{\lambda}$	Dimensionless perturbation scale $\hat{\lambda} = \mathcal{H}/k$
X	Kinetic energy: $X = -(1/2)g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$
Q	Coupling between a scalar field ϕ and non-relativistic matter
$\omega_{\rm BD}$	Brans–Dicke parameter
$R^2_{\rm GB}$	Gauss–Bonnet term

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