Analytic Number Theory
Essays in Honour of Klaus Roth

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Analytic Number Theory
Essays in Honour of Klaus Roth

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PREFACE

The fundamental contributions of Klaus Roth in analytic number theory have led to many breakthroughs over the past fifty years or so. His pioneering work on diophantine approximation, integer sequences, arithmetic progressions, irregularities of distribution, the large sieve and additive number theory has motivated successive generations of researchers in these areas.

On the occasion of Roth's eightieth birthday, we decided to compile a volume of essays on analytic number theory, written by a selection of his colleagues and friends, and with emphasis on his contributions and their substantial subsequent developments. In particular we have ensured that a substantial proportion of these essays are of a survey nature, for the benefit of experienced researchers and beginning research students alike. The remaining essays include some of the latest and exciting developments arising from Roth's earlier work.

W.W.L.C.
W.T.G.
H.H.
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R.C.V.

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The photograph of Klaus Roth is reproduced here with kind permission of the Royal Society and the Godfrey Argent Studio.

Lastly, but definitely not least, THANK YOU, KLAUS!

KLAUS ROTH AT 80

Heini Halberstam writes:

I first met Klaus Roth when he joined the Mathematics Department of University College London (UCL) as a graduate student in the autumn of 1946. He had spent the previous year in Scotland, teaching at Gordonstoun School and playing chess, and I think that it was in this period that he decided to commit himself in future to mathematics.

I too was embarking at this time on graduate studies in mathematics at UCL, and since we were of an age, taking much the same courses, and both of us were assigned to the supervision of Dr Theodor Estermann, we were naturally thrown into each other's company, especially as we came from a not dissimilar Central European background. For me this welcome companionship turned out to be an important formative experience; I came to understand what real talent looked like at close quarters, and I learned to live in its shadow – not always easy because, for one thing, our common advisor was as quick as I to notice the difference between us!

Our first year was enlivened by two splendid lecture courses, one on number theory by Harold Davenport who had just then joined the faculty at UCL, and the other by Hyman Kestelman, a former student of Estermann, on real analysis. In the second year we both wrote master's dissertations on topics mostly in the area of Waring's problem, proposed by Estermann. We had little difficulty with the mathematics, but preparing the manuscripts for submission in those pre-computer days was a nightmare! Our examiners for the orals were Miss G.K. Stanley, a student of G.H. Hardy from his Oxford days, and Estermann himself; and the orals were to be held at Westfield College where Miss Stanley taught. Since Roth's mother lived within walking distance, she entertained us for lunch beforehand. I recall having an attack of nerves, because I guessed that I would be asked to prove that

$$\sum_{n\leqslant x} d^2(n) \ll x\log^3 x,$$

$d(\,\cdot\,)$ being the divisor function. I told Klaus that I could do it if I knew that

$$d(mn) \leqslant d(m)d(n)$$

for all integer points (m,n), and he suggested at once that perhaps this was true! No sooner said than done; and Estermann professed himself pleased when I produced the argument at the viva – he said he hadn't thought to do it that way.

Actually Roth was by now well on his way to the PhD, and I think he started the academic year 1947–48 as an Assistant Lecturer at UCL. His progress from this point on was rapid; he soon came under the influence of Davenport, and new fields of endeavour opened up to him. I was his demonstrator, or teaching assistant, in that first year, and it was in this way that I came to meet a young woman in his intermediate class whom, he assured me, he intended to marry. It would take several years, but Klaus courted Melek Khaïry with single-minded devotion until he succeeded. Melek gained a PhD in psychology, they married and were inseparable for many years until her untimely death. Her passing in 2002 is a devastating blow to Klaus.

I left UCL in the summer of 1949 for a junior position at the then University College of the South West of England in Exeter, but Klaus and I remained in touch. We corresponded from time to time and he encouraged me, as he had done when I was preparing my doctoral thesis – looking back, he really acted as my de facto thesis advisor. On several occasions he arranged for me to give talks in Davenport's seminar and I found these occasions a valuable spur to mathematical activity. It was early in my time in Exeter that Klaus came down for a visit, and he reported then on his recent elegant work on gaps between square-free numbers. Apparently, when Klaus had talked about this subject in Davenport's seminar, Estermann had remarked that Klaus had been lucky to reduce the problem to the solution of a simple diophantine equation. Klaus did not believe that this was a lucky accident, and we set about examining the same problem for cube-free and fourth-power-free numbers. In each case we were able to reduce the corresponding argument to a similar diophantine resolution; and when Klaus returned to London he discovered a beautiful framework within which to resolve the general k-free problem.[1]

It was one summer, in 1951 I think, that Klaus and Melek stayed in our flat in Exeter for a week while we were away. I was preparing my thesis then and it was lying on a desk, in front of the window, the pages un-numbered. One of them opened the window and the wind scattered the wretched thesis all over the room. Klaus said afterwards that reassembling it was one of the cleverest things he had done that year!

In those early years Paul Erdős befriended us, and it was with his sustained encouragement – and advice – that we embarked eventually on writing *Sequences*. But of course, much had happened in London by then. Klaus and Anne Davenport had translated and revised Vinogradov's classic monograph on exponential sums.[2] Also, at the 1954 ICM in Amsterdam he reported on his influential work on irregularities of distribution, and not long afterwards came his seminal result about integer sequences having no three terms in arithmetic progression. To cap these achievements, when Davenport asked Klaus to prepare some lectures on work done on the Thue-Siegel theorem, Klaus actually proved the celebrated and long-desired form of this theorem that would forever add his name to theirs. Siegel himself declared this achievement as gaining Roth immortality, and in the short term it gained him a Fields Medal, the first for a British mathematician, at the 1958 ICM in Edinburgh.

By that time we had already begun work on *Sequences*, and when the manuscript was finished, Klaus carried it by hand to Oxford! It appeared in 1965 and was well received, due largely to Roth's contributions and his fame. In particular, he enjoyed writing about Rényi's version of the large sieve and he worked on this after the publication of the book until he came up with almost its best possible form. Writing about probabilistic aspects of sequences had been a learning experience for us both. There was a difficult quasi-independence result that we had needed in Chapter III, and I remember that Klaus managed to prove it on a weekend visit to Cambridge for a feast at his old college Peterhouse.

[1]Many years later, Mohan Nair gave an unexpected application to the distribution of k-free values of polynomials, and Michael Filaseta and Ognian Trifonov sharpened Roth's method significantly.

[2]To such good effect that in due course this translation was itself translated back into Russian!

Eventually we drifted apart, though we have remained in sporadic touch over the years. I welcome this opportunity to acknowledge my gratitude to Klaus Roth for the many years of our friendship and for his support in earlier years.

Wolfgang Schmidt writes:

I have always admired Professor Roth, or Klaus as he allowed me to call him. Some of my best work is based on his work on diophantine approximation and on irregularities of distribution. I appreciate his kindness, modesty and dry sense of humour.

In 1961/62, when I was quite young, I had an embarrassing experience. I had the honour of giving a talk at University College in London. I spoke about an extension to simultaneous approximation to two numbers of Roth's famous theorem on approximation to an algebraic number. I said that some part of the new work followed easily from Roth's ideas, and I wrote on the blackboard *easily modulo Roth*. The way I remember it, Roth, owing to some other obligations, arrived a little late. By then the *modulo* had somehow been erased. Roth understood that I certainly had had no intention of putting him down, and kindly teased me about my writing *easy Roth*.

A few years later Klaus and his gracious wife Melek spent a semester at my university in Colorado. It was a very enjoyable time. Somebody assigned him an office without windows, to be reached after crossing a machine room of the engineering school. The humming of machines was quite audible there. I asked him if we should not arrange another room, but to my surprise he said he liked the machine environment, and the humming was reassuring and drowned out other noises. I still do not know whether he liked that place or just did not want to give any trouble.

Unfortuntately I have not seen Klaus for a very long time.

Bob Vaughan writes:

My first recollection of Klaus was at a reception for new mathematics students at University College London in late 1963, my first year as an undergraduate. One of the third year students present pointed out Klaus with some awe and mentioned that he had solved a famous problem. Little did I realise then that I would eventually become his colleague. It was in the spring of 1971, when I was a Junior Research Fellow at Sheffield, that Klaus invited me to give a seminar at Imperial College London. When he asked after the seminar whether I was interested in applying for a position at Imperial College I said “yes” with alacrity.

Klaus is a very keen chess player having played top board for Cambridge while a student. His style of play is very spectacular and I remember the one occasion I saw him play was when the Mathematics Department staff played the students. Klaus had won his game before my game was out of the opening!

That part of my book on the Hardy-Littlewood method which describes Klaus' work on no three terms in arithmetic progression is, of course, based on the beautiful graduate lectures he gave on the subject in the late 1970s. I remember being very impressed by the lack of lecture notes, just a few lines written on one sheet of paper as an aide-memoir.

William Chen writes:

The first time I attended the undergraduate lectures on analytic number theory at Imperial College, Klaus Roth came into the room and promptly apologized to the class in advance for that inevitable "one of those days", easily recognizable when it happened. Well, it did not happen until towards the end of the course, during one of the lectures on Dirichlet's theorem on primes in arithmetic progression. After what seemed an eternity, he succeeded in writing some complicated term on the blackboard, and then retired to the back of the room. Some deep thought resulted in an equal sign, followed by precisely the same expression that had preceded it. We held our collective breath, wondering what would happen next. Klaus retired to the back of the room for more thought. A short time later, he approached the blackboard again and wrote down $+O(1)$, at which point we all burst into laughter. He spent a while admiring his wonderfully deep creation, turned round and said nonchalantly, "But this is correct, isn't it?"

In the same course, he challenged the class to recreate Besicovitch's very short proof, using the pigeon-hole principle, of the Erdős result that every rearrangement of the first $n^2 + 1$ natural numbers contains a monotonic subsequence of $n + 1$ terms. A bottle of beer was the prize, but the proof must not exceed a quarter of an A4 page. The following week, I gave him a constructive proof, also using the pigeon hole principle but still within the quarter page limit. He gave me a bottle of beer, but explained to the class that this was only a consolation prize, as Besicovitch's proof was non-constructive!

The following year, I began my PhD programme with Klaus as my official advisor.

In one of our first meetings, he explained to me that one must never let mathematics become the highest priority in life. He went on to say that for him, his wife Melek and ballroom dancing, for instance, came higher on his list of priorities.

He also impressed upon me the importance of numbering the pages whenever one wrote anything, although he did not give me a good reason for doing so.[3]

Klaus got me interested in irregularities of distribution very early on. In the late 1970s, he had written two very beautiful papers on upper bounds which showed that his lower bound in 1954 on the classical mean squares problem was best possible, and I was given preprints of both papers. The two papers were quite different, but both required very delicate constructions of point sets, followed by probabilistic arguments. In particular, the construction in the earlier of the two papers used multiple copies of the same two-dimensional lattice on top of each other, but each copy was carefully shifted horizontally. However, it was the dedicatory line on the manuscript that was most intriguing. After a search of the Mathematical Reviews, I concluded that the recipient of the dedication could not be a mathematician, but of course it was inappropriate for me to enquire further! It was years later that I accidentally found out that this was none other than the world ballroom dancing champion Alan Fletcher. When I told Klaus of my unusual discovery, he explained that at the time he was working on the problem and failing to solve it, he and Melek had regular dancing lessons from Fletcher who noticed that something was clearly wrong. Upon finding out the cause, Fletcher proceeded to ask Klaus every week whether he had solved the problem. So eventually the problem was solved and the dancing improved. Klaus commented cheekily that there was no other way of "getting him off my back"!

In fact, it was a generalization of the results in these two papers to higher moments that formed the first steps of my career.

[3]See Heini Halberstam's account!

For many years, Klaus loved to work late at night. Early on, he gave me his home phone number. I could phone him if I had difficulties making progress and could not wait until our next scheduled meeting. No time in the night would be considered *too late*, but there was definitely something called *too early*. Thus while it would be perfectly in order for me to call him at two in the morning, I would be well advised to leave him alone at ten in the morning. Years later, when he discovered that I preferred to lecture early in the morning, he commented that he had not trained me very well.

Indeed, Klaus is a very interesting colleague, and he loves playing the odd practical joke on others. One day in late 1983, he stormed into my office and exclaimed, "How dare you give me notes that are out of date!" I had given him a set of notes on elementary number theory which I had used the previous year and which included a proof of the famous four-squares theorem. In the introduction, I had included the "out-of-date" example

$$1982 = 40^2 + 15^2 + 11^2 + 6^2.$$

Luckily, I detected that he was trying very hard to suppress a cheeky smile on his face. By a stroke of good luck, I had given a popular lecture in early 1983, and the "up-to-date" example was soon retrieved from my filing cabinet.

On another occasion, he called me to his office. He was having terrible difficulties adding the marks in the examination scripts, and had borrowed a calculator from another colleague to help him. The only problem was that he could not turn the pages and enter the numbers at the same time, so needed me to read the marks to him. Progress was painfully slow. At one point, I realized that one of the students had done very badly indeed. Nevertheless, I read the marks to Klaus very slowly, "Zero, plus, zero, plus, zero, plus, zero, plus, six, plus, zero, equals." He entered each command carefully, and then concluded with the cheerful remark, "Oh, I could have done this one by theory!"

Klaus is not just an advisor and colleague. Over the years, he and Melek have become our very good friends. I recall the occasion when I told him that I was planning to get married, and he remarked, "So Melek's efforts would not be wasted." On one occasion, he and Melek saw an extremely elegant coffee set in a shop. Melek was convinced that it would make the ideal wedding present for me, and they bought it straightaway – years before I met my wife!

Dinners at the Roth household were wonderful occasions. Melek cooked beautifully, and Klaus served wine in great style. Unfortunately, my attempt to teach Klaus to use the chopsticks was an abject failure. Clearly I was unable to convince him that he was facing a two-dimensional problem in three-dimensional euclidean space.

After Klaus retired, he and Melek moved to Inverness where they lived in a beautiful house on the hill and with a wonderful view of the river below. However, Klaus is most proud of the big living room downstairs which they converted into a dance room. No doubt, he and Melek spent many happy hours there perfecting their steps. We visited them on quite a few occasions, even after our translation to Australia, and we always had a wonderful time whenever we went there. Melek made our children very welcome and went out of her way to look after them.

On one of these visits to Inverness, just before we said good-bye, Melek asked me to promise her that I would take great care of my wife and children. I did not realize then that I was not to see her again. She has been the strength behind Klaus, and her

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passing is a great blow to him. Remarkably he soldiers on, and we chat fairly regularly on the phone.

This project of essays in honour of Klaus began some time ago without his knowledge. I told him about it some time in 2007. He wishes to thank everyone for their contributions. I am sure we all wish him well, and hope that this volume will give him some pleasure.