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978-0-521-51407-1 - Numerical Relativity: Solving Einstein's Equations on the Computer

Thomas W. Baumgarte and Stuart L. Shapiro

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