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978-0-521-49922-4 - Lectures on Lie Groups and Lie Algebras

Roger Carter, Graeme Segal and Ian Macdonald

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