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978-0-521-49922-4 - Lectures on Lie Groups and Lie Algebras

Roger Carter, Graeme Segal and Ian Macdonald

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Foreword

This book consists of notes based on the three introductory lecture courses given at the LMS-SERC Instructional Conference on Lie theory and algebraic groups held at Lancaster University in September 1993: Lie Algebras by Roger Carter; Lie Groups by Graeme Segal; algebraic groups by Ian Macdonald.

The aim of the course was to provide an introduction to this important area of mathematics for postgraduate students who had no previous specialised knowledge. Discussions with the students at the end of the meeting suggested that the conference had been extremely successful; it then seemed desirable to further impose on the lecturers by asking them to write-up their lectures, in order that future generations of students could also benefit from their efforts.

All three lecturers adopted the same approach of providing a crisp, fast-moving, clear introduction, while at the same time taking care to indicate more advanced material, so as to give the full flavour of the subject. It is clear, from both the lectures and the written account, that a substantial effort was made to ensure a coherent and well-harmonised presentation of these three highly interrelated themes.

The general intention of the new series of LMS-SERC Instructional Conferences is to provide postgraduate students with the opportunity to learn important mainstream core mathematics, which they might not otherwise meet. Lie theory and algebraic groups seemed to be a natural first choice, since they are a central mathematical crossroads, which relate to a host of important areas such as group theory, number theory, algebraic geometry, differential geometry, topology, particle physics and strings; indeed, a knowledge of algebraic groups and Lie theory can be quite crucial in making significant progress in many aspects of these related areas.

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Finally, on behalf of the LMS, I should like to express my deepest gratitude to the three authors for not just accepting to give their lectures and then write them up, but also for carrying out their allotted tasks with such infectious enthusiasm; extra special thanks go to Ian Macdonald for sage advice in the initial planning of the meeting. It is also a pleasure to thank both Roger Astley and David Tranah for their help and cooperation in producing these notes, which will be a very valuable contribution to the mathematical community.

Martin Taylor
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