Conference Programme

Oberwolfach 1993

Novikov Conjectures, Index Theorems and Rigidity

Monday, 6th September

9.30–10.30	A. S. Mishchenko	Analytic torsion over C^*-algebras
11.00–12.00	J. Roe	Coarse geometry and index theory
16.00–17.00	G. Kasparov	Groups acting on 'bolic' spaces and the Novikov conjecture
17.15–18.15	S. Weinberger	Coarse geometry and the Novikov conjecture

Informal evening session

20.00–20.30	A. Ranicki	Algebraic Novikov for analysts
20.45–21.15	J. Rosenberg	Analytic Novikov for topologists

Tuesday, 7th September

9.30–10.30	E. K. Pedersen	Controlled algebra and the Novikov conjecture
11.00–12.00	J. Eichhorn	Rigidity of the index on open manifolds
16.00–16.30	V. Mathai	Homotopy invariance of spectral flow and η-invariants
16.40–17.10	J. Kaminker	Duality for dynamical system-C^*-algebras and the Novikov conjecture
17.20–17.50	T. Koźniewski	On the Nil and UNil groups
18.00–18.30	S. Prassidis	K-theory rigidity of virtually nilpotent groups

Informal evening session

20.00–	S. Ferry (chair)	Various proofs of Novikov's theorem on the topological invariance of the rational Pontrjagin classes

Wednesday, 8th September

9.30–10.00	O. Attie	The Borel conjecture for manifolds with bounded geometry
10.15–10.45	E. Troitsky	The homotopy invariance of some higher signatures
11.00–11.30	P. Julg	The KK-ring and Baum-Connes conjecture for complex hyperbolic groups
11.45–12.15	W. Lück	L^2-Betti numbers and the Novikov-Shubin invariants
14.00–15.00	M. Gromov	Reflections on the Novikov conjecture
16.30–17.15	S. Hurder	Exotic index theory and the Novikov conjecture
17.30–18.15	B. Williams	A proof of a conjecture of Lott

Thursday, 9th September

9.30–10.30	M. Puschnigg	Cyclic homology and the Novikov conjecture
11.00–11.30	U. Bunke	Glueing problems for indices and the η-invariant
11.45–12.15	M. Yan	The role of signature in periodicity

Informal evening session

20.00–		**Problem session**

Friday, 10th September

9.30–10.00	R. Jung	Elliptic homology and the Novikov conjecture
10.15–10.45	C. Stark	Approximate finiteness properties
11.00–12.00	J. Cuntz	On excision in cyclic homology
14.00–14.30	F. X. Connolly	Ends of G-manifolds and stratified spaces
14.40–15.10	J. Miller	Signature operators and surgery groups

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Excerpt
More information

Participant List

Oberwolfach 1993

Novikov Conjectures, Index Theorems and Rigidity

Note: Affiliations are as of the time of the conference. Addresses are as current as possible as of June, 1995.

Organizers:

Ferry, Steve, SUNY at Binghamton
mail: Dept. of Mathematical Sciences, SUNY at Binghamton, Binghamton, NY 13901, U.S.A.
email: steve@math.binghamton.edu

Ranicki, Andrew, University of Edinburgh
mail: Dept. of Mathematics and Statistics, James Clerk Maxwell Bldg., University of Edinburgh, Edinburgh EH9 3JZ, Scotland, U.K.
email: a.ranicki@edinburgh.ac.uk

Rosenberg, Jonathan, University of Maryland
mail: Dept. of Mathematics, University of Maryland, College Park, MD 20742, U.S.A.
email: jmr@math.umd.edu

Other participants:

Attie, Oliver, McMaster University
mail: Dept. of Mathematics, University of California at Los Angeles, Los Angeles, CA 90024, U.S.A.
email: oattie@math.ucla.edu

Block, Jonathan, University of Pennsylvania
mail: Dept. of Mathematics, University of Pennsylvania, Philadelphia, PA 19104, U.S.A.
email: blockj@archimedes.math.upenn.edu

Bunke, Ulrich, Humboldt University
mail: Dept. of Mathematics, Humboldt Universität, Unter den Linden 6, 10099 Berlin, Germany.
email: bunke@mathematik.hu-berlin.de

Connolly, Frank, University of Notre Dame
mail: Dept. of Mathematics, University of Notre Dame, Notre Dame, IN 46556, U.S.A.
email: francis.x.connolly.1@math.nd.edu

Cuntz, Joachim, Heidelberg University
mail: Mathematisches Institut, Universität Heidelberg, Im Neuenheimer Feld 288, 69120 Heidelberg, Germany.
email: `cuntz@math.uni-heidelberg.de`

Davis, James, Indiana University
mail: Dept. of Mathematics, Indiana University, Bloomington, IN 47405, U.S.A.
email: `jfdavis@ucs.indiana.edu`

tom Dieck, Tammo, Göttingen University
mail: Mathematisches Institut, Universität Göttingen, Bunsenstraße 3–5, 37073 Göttingen, Germany.
email: `tammo@cfgauss.uni-math.gwdg.de`

Eichhorn, Jürgen, Greifswald University
mail: Fachbereich Mathematik, Universität Greifswald, Jahnstraße 15a, 17487 Greifswald, Germany.
email: `eichhorn@math-inf.uni-greifswald.dbp.de`

Gromov, Misha, I.H.E.S.
mail: Institut des Hautes Études Scientifiques, 35 route de Chartres, 91440 Bures-sur-Yvette, France.
email: `gromov@ihes.fr; gromov@math.umd.edu`

Hughes, Bruce, Vanderbilt University
mail: Dept. of Mathematics, Vanderbilt University, Nashville, TN 37240, U.S.A.
email: `hughescb@athena.cas.vanderbilt.edu`

Hurder, Steve, University of Illinois at Chicago
mail: Dept. of Mathematics (m/c 249), University of Illinois at Chicago, 851 S. Morgan St., Chicago, IL 60607, U.S.A.
email: `hurder@boss.math.uic.edu`

Julg, Pierre, Louis Pasteur University
mail: Institut de Recherche Mathématique Avancée, Université Louis Pasteur, 7 rue René Descartes, 67084 Strasbourg Cedex, France.
email: `julg@math.u-strsbg.fr`

Jung, Rainer, University of Mainz
mail: Fachbereich Mathematik, Johannes Gutenberg Universität Mainz, 55099 Mainz, Germany.
email: `jung@topologie.mathematik.uni-mainz.de`

Kaminker, Jerry, Indiana/Purdue University at Indianapolis
mail: Dept. of Mathematics, Indiana Univ./Purdue Univ. at Indianapolis, 402 N. Blackford Street, Indianapolis, IN 46202, U.S.A.
email: `kaminker@math.iupui.edu`

Kasparov, Gennadi, University of Marseille–Luminy
mail: Département Mathématique–Informatique, Université de Marseille–Luminy, 13288 Marseille Cedex 9, France.
email: kasparov@lumimath.univ-mrs.fr

Kassel, Christian, Louis Pasteur University
mail: Institut de Recherche Mathématique Avancée, Université Louis Pasteur, 7 rue René Descartes, 67084 Strasbourg Cedex, France.
email: kassel@math.u-strasbg.fr

Koźniewski, Tadeusz, University of Warsaw
mail: Inst. of Mathematics, Uniwersytet Warszawski, 00–901 Warszawa, Poland.
email: tkozn@mimuw.edu.pl

Lesch, Matthias, Augsburg University
mail: Mathematisches Institut, Universität Augsburg, Universitätsstraße 14, 86135 Augsburg, Germany.
email: lesch@uni-augsburg.de

Lück, Wolfgang, Mainz University
mail: Fachbereich Mathematik, Johannes Gutenberg Universität Mainz, 55099 Mainz, Germany.
email: lueck@topologie.mathematik.uni-mainz.de

Luke, Glenys, Oxford University
mail: St. Hugh's College, Oxford OX2 6LE, England, U.K.
email: glenys.luke@st_hugh.ox.ac.uk

Mathai, Varghese, University of Adelaide
mail: Dept. of Pure Mathematics, University of Adelaide, Adelaide, SA 5005, Australia.
email: vmathai@maths.adelaide.edu.au

Mavra, Boris, Oxford University
mail: Mathematical Institute, Oxford University, 24–29 St. Giles', Oxford OX1 3LB, England, U.K.
email: mavra@maths.ox.ac.uk

Miller, John, Indiana/Purdue University at Indianapolis
mail: Dept. of Mathematics, Indiana Univ./Purdue Univ. at Indianapolis, 402 N. Blackford Street, Indianapolis, IN 46202, U.S.A.
email: jmiller@math.iupui.edu

Mishchenko, Alexander, Moscow State University
mail: Dept. of Mathematics and Mechanics, Moscow State University, Moscow 119899, Russia.
email: asmish@mech.math.msu.su

Pedersen, Erik, SUNY at Binghamton
mail: Dept. of Mathematical Sciences, SUNY at Binghamton, Binghamton, NY 13901, U.S.A.
email: erik@math.binghamton.edu

Prassidis, Stratos, Vanderbilt University
mail: Dept. of Mathematics, Vanderbilt University, Nashville, TN 37240, U.S.A.
email: prassie@athena.cas.vanderbilt.edu

Puschnigg, Michael, Heidelberg University
mail: Mathematisches Institut, Universität Heidelberg, Im Neuenheimer Feld 288, 69120 Heidelberg, Germany.
email: puschnig@vogon.mathi.uni-heidelberg.de

Roe, John, Oxford University
mail: Jesus College, Oxford OX1 3DW, England, U.K.
email: jroe@spinoza.jesus.ox.ac.uk

Rothenberg, Mel, University of Chicago
mail: Dept. of Mathematics, University of Chicago, Chicago IL 60637, U.S.A.
email: mel@math.uchicago.edu

Stark, Chris, University of Florida
mail: Dept. of Mathematics, University of Florida, P. O. Box 11800, Gainesville, FL 32611, U.S.A.
email: cws@math.ufl.edu

Troitsky, Evgenii, Moscow State University
mail: Dept. of Mathematics and Mechanics, Moscow State University, Moscow 119899, Russia.
email: troitsky@mech.math.msu.su

Valette, Alain, University of Neuchâtel
mail: Institut de Mathématiques, Université de Neuchâtel, Chantemerle 20, CH-2007 Neuchâtel, Switzerland.
email: valette@maths.unine.ch

Weinberger, Shmuel, University of Chicago
mail: Dept. of Mathematics, University of Pennsylvania, Philadelphia, PA 19104, U.S.A.
email: shmuel@archimedes.math.upenn.edu

Williams, Bruce, University of Notre Dame
mail: Dept. of Mathematics, University of Notre Dame, Notre Dame, IN 46556, U.S.A.
email: bruce@bruce.math.nd.edu

Yan, Min, Hong Kong University of Science and Technology
mail: Dept. of Mathematics, Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong.
email: mamyan@usthk.ust.hk

A History and Survey of the Novikov Conjecture

Steven C. Ferry, Andrew Ranicki, and Jonathan Rosenberg

Contents

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1. Precursors of the Novikov Conjecture

Characteristic classes

The Novikov Conjecture has to do with the question of the relationship of the characteristic classes of manifolds to the underlying bordism and homotopy theory. For smooth manifolds, the characteristic classes are by definition the characteristic classes of the tangent (or normal) bundle, so basic to this question is another more fundamental one: how much of a vector bundle is determined by its underlying spherical fibration? The Stiefel-Whitney classes of vector bundles are invariants of the underlying spherical fibration, and so the Stiefel-Whitney numbers of manifolds are homotopy invariants. Furthermore, they determine unoriented bordism. The Pontrjagin classes of vector bundles are not invariants of the underlying spherical fibration, and the Pontrjagin numbers of manifolds are not homotopy invariants. However, together with the Stiefel-Whitney numbers, they do determine oriented bordism. The essential connection between characteristic numbers and bordism was established by Thom [Th1] in the early 1950's.

Geometric rigidity

As we shall see later, the Novikov Conjecture is also closely linked to problems about rigidity of aspherical manifolds. As everyone learns in a first course in geometric topology, closed 2-manifolds are determined up to homeomorphism by their fundamental groups. In higher dimensions, of course, nothing like this is true in general, but one can still ask if *aspherical* closed manifolds (closed manifolds having contractible universal cover) are determined up to homeomorphism by their fundamental groups. That this should be the case is the *Borel Conjecture* formulated by Armand Borel in the 50's (according to Hsiang in [Hs3]), and communicated to various people in the 60's. In dimension 2, restricting attention to aspherical manifolds is little loss of generality, since S^2 and $\mathbb{RP}^2$ are the only closed 2-manifolds which are *not* aspherical. The Mostow Rigidity Theorem was the most dramatic early evidence for the Borel Conjecture, proving the conjecture for closed manifolds which are locally symmetric spaces.

The Hirzebruch signature theorem

The actual history of the Novikov Conjecture starts with the Hirzebruch signature theorem [Hir], which expresses the signature of an oriented closed $4k$-dimensional manifold M in terms of characteristic classes:

$$\text{signature}(M) \;=\; \langle \mathcal{L}(M),\, [M] \rangle \in \mathbb{Z}\,.$$

Here, $\mathcal{L}(M) \in H^{4*}(M;\mathbb{Q})$ is the $\mathcal{L}$-class of M, a certain formal power series in the Pontrjagin classes $p_*(M) \in H^{4*}(M)$ with rational coefficients. The formula is surprising in that the left hand side is an integer which only depends on the structure of the cohomology ring of M, whereas the right hand side is a sum of rational numbers which are defined (at least *a priori*) in terms of the differentiable structure. The (inhomogeneous) class $\mathcal{L}(M)$ determines all of the rational Pontrjagin classes of M, but only the component of $\mathcal{L}(M)$ in the dimension of M is homotopy invariant – in fact, the other components are not even bordism invariants. Milnor [Miln1] used the signature theorem to verify that the homotopy spheres he constructed do indeed have exotic differentiable structures.

The converse of the signature theorem (Browder, Novikov)

Following the development of Thom's bordism theory, Milnor [Miln2] proved that two manifolds are bordant if and only if they are related by a finite sequence of surgeries. This was the beginning of the use of surgery as a fundamental tool in differential topology. Soon afterwards, Kervaire and Milnor [KerM] used surgery to classify exotic spheres in dimensions ≥ 7. In 1962, Browder ([Br1],[Br4]) and Novikov [Nov1], working independently, applied the same technique to manifolds with more complicated homology. They used surgery theory to establish a converse to the Hirzebruch signature theorem in dimensions ≥ 5: if X is a simply-connected $4k$-dimensional Poincaré space, such that the signature of X is the evaluation on the fundamental class $[X] \in H_{4k}(X)$ of $\mathcal{L}(-\nu)$ for some vector bundle ν with spherical Thom class, then X is homotopy equivalent to a smooth closed manifold M with stable normal bundle pulled back from ν. A consequence of this is that for simply-connected $4k$-dimensional manifolds in high dimensions, the top degree term of the $\mathcal{L}$-class is essentially the *only* homotopy-invariant rational characteristic class. Novikov ([Nov1], [Nov2], [Nov3]) extended these ideas to the study of the *uniqueness* properties of manifold structures within a homotopy type. Sullivan [Sul] then combined the Browder-Novikov surgery theory with homotopy theory to reformulate the surgery classification of manifolds in terms of the *surgery exact sequence* of pointed sets, which for a $4k$-dimensional simply-connected manifold M $(k > 1)$ has the form:

$$0 \to \mathcal{S}(M) \xrightarrow{\theta} [M, G/O] \xrightarrow{A} \mathbb{Z}$$

with $\mathcal{S}(M)$ the *structure set* of M, consisting of the equivalence classes of pairs (N, f) with N a closed manifold and $f : N \to M$ a homotopy equivalence. Two such pairs (N, f), (N', f') are equivalent if there exists a diffeomorphism $g : N \to N'$ with a homotopy $f'g \simeq f : N \to M$. Here G/O is the homotopy fiber of the forgetful map J from the classifying space BO for stable vector bundles to the classifying space BG for stable spherical fibrations, and the map θ sends an element (N, f) of the structure set to the difference between the stable normal bundle of M and the push-forward under f of the stable normal bundle of N. (Both are lifts of the same underlying spherical fibration, the Spivak normal fibration [Spv].) The map A sends an element of $[M, G/O]$, represented by a vector bundle η over M with a fiber homotopy trivialization, to

$$\langle \mathcal{L}(\tau_M \oplus \eta) - \mathcal{L}(\tau_M), [M] \rangle,$$

where τ_M is the tangent bundle of M.