These lecture notes are intended as a non-technical overview of scattering theory. The point of view adopted throughout is that scattering theory provides a parametrization of the continuous spectrum of an elliptic operator on a complete manifold with uniform structure at infinity. The simple and fundamental case of the Laplacian on Euclidean space is described in the first two lectures to introduce the basic framework of scattering theory. In the next three lectures various results on Euclidean scattering, and the methods used to prove them, are outlined. In the last three lectures these ideas are extended to non-Euclidean settings.

These lecture notes will be of interest to researchers and graduate students in analysis and differential geometry.

Geometric scattering theory

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CAMBRIDGE
UNIVERSITY PRESS

Published by the Press Syndicate of the University of Cambridge
The Pitt Building, Trumpington Street, Cambridge CB2 1RP
40 West 20th Street, New York, NY 10011–4211, USA
10 Stamford Road, Oakleigh, Melbourne 3166, Australia

First Published 1995

Library of Congress cataloging-in-publication data available

A catalogue record for this book is available from the British Library

ISBN 0–521–49673–X hardback
ISBN 0–521–49810–4 paperback

Transferred to digital printing 2003

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Preface

These notes are based on lectures delivered at Stanford University in January[1] 1994 and then repeated at MIT in the Spring semester. I am very grateful to the members of the Mathematics Department at Stanford, and in particular Ralph Cohen, for the invitation and hospitality. My especial thanks to those who attended the lectures and contributed in one way or another. I am particularly pleased to acknowledge the influence on my thinking of two of the members of the audience, Ralph Phillips and Joe Keller. Rafe Mazzeo encouraged me to write up the lectures, provided me with his own notes and, as if that were not enough, made many helpful comments on the manuscript. I should also like to extend my thanks to Sang Chin, Daniel Grieser, Andrew Hassell, Mark Joshi, Olivier Lafitte, Eckhard Meinrenken, Edith Mooers and Andras Vasy, who attended the second hearing[2] of the lectures at MIT and together made many useful remarks; Andras Vasy was particularly helpful in reading and correcting the notes as they dribbled out. I would also like to thank Tanya Christiansen and Gunther Uhlmann for their assistance and Lars Hörmander, Georgi Vodev and Maciej Zworski for their comments on later versions of the manuscript.[3]

It is my hope that these notes may serve as an introduction to an active and growing area or research, although I fear they represent a rather steep learning curve.

[1] It was a horrible month in Cambridge I am told, very pleasant indeed in Palo Alto. This footnote is an indication of things to come in the body of the notes. If you can't stand it, stop now!

[2] Of course I had really wanted to do things in the other order but did not manage to get my thoughts together in time.

[3] Of course, I claim sole credit for all remaining errors.

Introduction

The lectures on which these notes are based were intended as an, essentially non-technical, overview of scattering theory. The point of view adopted throughout is that scattering theory provides a parametrization of the continuous spectrum of an elliptic operator on a complete manifold with uniform structure at infinity. The simple, and fundamental, case of the Laplacian on Euclidean space is described in the first two lectures to introduce the basic framework of scattering theory. In the next three lectures various results on Euclidean scattering, and the methods used to prove them, are outlined. In the last three lectures these ideas are extended to non-Euclidean settings. This is an area of much current research and my idea was to show how similar the Euclidian and the less familiar cases are. Some of the interactions of scattering theory with hyperbolic geometry, index theory and Hodge theory are also indicated.

I have made no attempt at completeness here but simply described what time, and my own tastes, indicate. In particular there should be at least three times as many references as there are. If I have offended by omitting reference to important work, this should not be interpreted as a deliberate slight! In writing up the lectures I have made extensive use of footnotes to cover more subtle points, to clarify statements that were felt to be obscure, by someone, and to make comments. These asides can be freely ignored.