1 Mechanical vibrations: a review of some fundamentals

1.1 Introduction

Noise and vibration are often treated separately in the study of dynamics, and it is sometimes forgotten that the two are inter-related – i.e. they simply relate to the transfer of molecular motional energy in different media (generally fluids and solids respectively). It is the intention of this book to bring noise and vibration together within a single volume instead of treating each topic in isolation. Central to this is the concept of wave–mode duality; it is generally convenient for engineers to think of noise in terms of waves and to think of vibration in terms of modes. A fundamental understanding of noise, vibration and interactions between the two therefore requires one to be able to think in terms of waves and also in terms of modes of vibration.

This chapter reviews the fundamentals of vibrating mechanical systems with reference to both wave and mode concepts since the dynamics of mechanical vibrations can be studied in terms of either. Vibration deals (as does noise) with the oscillatory behaviour of bodies. For this oscillatory motion to exist, a body must possess inertia and elasticity. Inertia permits an element within the body to transfer momentum to adjacent elements and is related to density. Elasticity is the property that exerts a force on a displaced element, tending to return it to its equilibrium position. (Noise therefore relates to oscillatory motion in fluids whilst vibration relates to oscillatory motion in solids.)

Oscillating systems can be treated as being either linear or non-linear. For a linear system, there is a direct relationship between cause and effect and the principle of superposition holds – i.e., if the force input doubles, the output response doubles. The relationship between cause and effect is no longer proportional for a non-linear system. Here, the system properties depend upon the dependent variables, e.g. the stiffness of a non-linear structure depends upon its displacement.

In this book, only linear oscillating systems which are described by linear differential equations will be considered. Linear system analysis adequately explains the behaviour of oscillatory systems provided that the amplitudes of the oscillations are very small relative to the system's physical dimensions. In each case, the system (possessing inertia and elasticity) is initially or continuously excited in the presence of external forces

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which tend to return it to its undisturbed position. Noise levels of up to about 140 dB (\sim 25 m from a jet aircraft at take off) are produced by linear pressure fluctuations. Most engineering and industrial type noise sources (which are generally less than 140 dB) and the associated mechanical vibrations can therefore be assumed to behave in a linear manner. Some typical examples are the noise and vibration characteristics of industrial machinery, noise and vibration generated from high speed gas flows in pipelines, and noise and vibration in motor vehicles.

The vibrations of linear systems fall into two categories - free and forced. Free vibrations occur when a system vibrates in the absence of any externally applied forces (i.e. the externally applied force is removed and the system vibrates under the action of internal forces). A finite system undergoing free vibrations will vibrate in one or more of a series of specific patterns: for instance, consider the elementary case of a stretched string which is struck at a chosen point. Each of these specific vibration patterns is called a mode shape and it vibrates at a constant frequency, which is called a natural frequency. These natural frequencies are properties of the finite system itself and are related to its mass and stiffness (inertia and elasticity). It is interesting to note that if a system were infinite it would be able to vibrate freely at any frequency (this point is relevant to the propagation of sound waves). Forced vibrations, on the other hand, take place under the excitation of external forces. These excitation forces may be classified as being (i) harmonic, (ii) periodic, (iii) non-periodic (pulse or transient), or (iv) stochastic (random). Forced vibrations occur at the excitation frequencies, and it is important to note that these frequencies are arbitrary and therefore independent of the natural frequencies of the system. The phenomenon of resonance is encountered when a natural frequency of the system coincides with one of the exciting frequencies. The concepts of natural frequencies, modes of vibration, forced vibrations and resonance will be dealt with later on in this chapter, both from an elastic continuum viewpoint and from a macroscopic viewpoint.

The concept of damping is also very important in the study of noise and vibration. Energy within a system is dissipated by friction, heat losses and other resistances, and any damped free vibration will therefore diminish with time. Steady-state forced vibrations can be maintained at a specific vibrational amplitude because the required energy is supplied by some external excitation force. At resonance, it is only the damping within a system which limits vibrational amplitudes. Both solids and fluids possess damping, and the response of a practical system (for example, a built-up plate or shell structure) to a sound field is dependent upon both structural damping and acoustic radiation damping. The concepts of structural damping will be introduced in this chapter and discussed in more detail in chapter 6 together with acoustic radiation damping.

A macroscopic (modal) analysis of the dynamics of any finite system requires an understanding of the concept of degrees of freedom. The degrees of freedom of a system are defined as the minimum number of independent co-ordinates required to describe its motion completely. An independent particle in space will have three degrees of freedom,

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a finite rigid body will have six degrees of freedom (three position components and three angles specifying its orientation), and a continuous elastic body will have an infinite number of degrees of freedom (three for each point in the body). There is also a one to one relationship between the number of degrees of freedom and the natural frequencies (or modes of vibration) of a system – a system with p degrees of freedom will have p natural frequencies and p modes of vibration. Plates, shell and acoustic volumes, for instance, have many thousands of degrees of freedom (and therefore natural frequencies/modes of vibration) within the audible frequency range. As far as mechanical vibrations of structures (shafts, machine tools, etc.) are concerned, certain parts of the structures can often be assumed to be rigid, and the system can therefore be reduced to one which is dynamically equivalent to one with a finite number of degrees of freedom. Many mechanical vibration problems can thus be reduced to systems with one or two degrees of freedom.

An engineering description of the time response of vibrating systems can be obtained by solving linear differential equations based upon mathematical models of various equivalent systems. When a finite-number-of-degrees-of-freedom model is used, the system is referred to as a lumped-parameter system. Here, the real system is approximated by a series of rigid masses, springs and dampers. When an infinite-numberof-degrees-of-freedom model is used, the system is referred to as a continuous or a distributed-parameter system. The differential equation governing the motion of the structure is still the same as for the lumped-parameter system except that the mass, damping and stiffness distributions are now continuous and a wave-type solution to the equations can therefore be obtained. This wave-mode duality which is central to the study of noise and vibration will be discussed in some detail at the end of this chapter.

1.2 Introductory wave motion concepts – an elastic continuum viewpoint

A wave motion can be described as a phenomenon by which a particle is disturbed such that it collides with adjacent particles and imparts momentum to them. After collision, the particles oscillate about their equilibrium positions without advancing in any particular direction, i.e. there is no nett transport of the particles in the medium. The disturbance, however, propagates through the medium at a speed which is characteristic of the medium, the kinematics of the disturbance, and any external body forces on the medium. Wave motion can be described by using either molecular or particulate models. The molecular model is complex and cumbersome, and the particulate model is the preference for noise and vibration analysis. A particle is a volume element which is large enough to contain millions of molecules such that it is considered to be a continuous medium, yet small enough such that its thermodynamic and acoustic variables are constant. Solids can store energy in shear and compression, hence several

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types of waves are possible, i.e. compressional (longitudinal) waves, flexural (transverse or bending) waves, shear waves and torsional waves. Fluids, on the other hand, can only store energy in compression. Wave motion is simply a balance between potential and kinetic energies, with the potential energy being stored in different forms for different wave-types. Compressional waves store potential energy in longitudinal strain, and flexural waves store it in bending strain.

Some elementary examples of wave motion are the propagation of sound in the atmosphere due to a source such as blast noise from a quarry, bending motions in a metal plate (such as a machine cover) which is mechanically excited, and ripples in a moving stream of water due to a pebble being thrown into it. In the case of the sound radiation associated with the blasting process at the quarry, the waves that are generated would travel both upwind and downwind. Likewise, the ripples in the stream would also travel upstream and downstream. In both these examples the disturbances propagate away from the source without being reflected. For the case of the finite metal plate, a series of standing waves would be established because of wave reflection at the boundaries. In each of the three examples there is, however, no nett transport of mass particles in the medium.

It is important to note at this stage that it is mathematically convenient to model the more general time-varying wave motions that are encountered in real life in terms of summations of numerous single frequency (harmonic) waves. The discussions in this book will therefore relate to such models. The properties of the main types of wave motions encountered in fluids and solids are now summarised. Firstly, there are two different velocities associated with each type of harmonic wave motion. They are: (i) the velocity at which the disturbance propagates through the medium (this velocity is characteristic of the properties of the medium, the kinematics of the disturbance, and any external body forces on the medium), and (ii) the velocity of the oscillating mass particles in the medium (this particle velocity is a measure of the amplitude of the disturbance which produces the oscillation, and relates to the vibration or sound pressure level that is measured). These two types of velocities which are associated with harmonic waves are illustrated in Figure 1.1 for the case of compressional and flexural wave motions on an arbitrary free surface. For the compressional (longitudinal) wave, there are alternate regions of expansion and compression of the mass particles, and the particle and wave velocities are in the same direction. The propagation of sound waves in air and longitudinal waves in bars is typical of such waves. For the flexural (transverse or bending) wave, the particle velocity is perpendicular to the direction of wave propagation. The bending motion of strings, beams, plates and shells is typical of this type of wave motion. It will be shown later on (in chapter 3) that bending waves are the only type of structural waves that contribute directly to noise radiation and transmission through structures (e.g. aircraft fuselages). The main reason for this is that the particle velocity (and structural displacement) is perpendicular to the direction of wave propagation, as illustrated in Figure 1.1(b). This produces an effective disturbance of the adjacent fluid particles and results in an effective exchange of energy between





(a) Compressional (longitudinal) wave



(b) Flexural (transverse or bending) wave

Fig. 1.1. Illustration of wave and particle velocities.

the structure and the fluid. It will also be shown in chapter 3 that the bending wave velocity varies with frequency whereas other types of wave velocities (compressional, torsional, etc.) do not.

Any wave motion can be represented as a function of time, of space or of both. Time variations in a harmonic wave motion can be represented by the radian (circular) frequency ω . This parameter represents the phase change per unit increase of time, and

$$\omega = 2\pi/T,\tag{1.1}$$

where T is the temporal period of the wave motion. This relationship is illustrated in Figure 1.2. The phase of a wave (at a given point in time) is simply the time shift relative to its initial position. Spatial variations in such a wave motion are represented by the phase change per unit increase of distance. This parameter is called the wavenumber, k, where

$$k = \omega/c$$
,

(1.2)





Fig. 1.2. Time variations for a simple wave motion.



Fig. 1.3. Spatial variations for a simple wave motion.

and *c* is the wave velocity (the velocity at which the disturbance propagates through the medium). This wave velocity is also sometimes called the phase velocity of the wave – it is the ratio of the phase change per unit increase of time to the phase change per unit increase of distance. Now, the spatial period of a harmonic wave motion is described by its wavelegth, λ , such that

$$k = 2\pi/\lambda. \tag{1.3}$$

This relationship is illustrated in Figure 1.3, and the analogy between radian frequency, ω , and wavenumber, k, can be observed.

If the wave velocity, c, of an arbitrary time-varying wave motion (a summation of numerous harmonic waves) is constant for a given medium, then the relationship



Fig. 1.4. Linear and non-linear dispersion relationships.

between ω and k is linear and therefore non-dispersive – i.e. the spatial form of the wave does not change with time. On the other hand, if the wave velocity, c, is not constant (i.e. it varies with frequency), the spatial form of the wave changes with time and is therefore dispersive. It is a relatively straightforward exercise to show that a single frequency wave is non-dispersive but that a combination of several waves of different frequencies is dispersive if they each propagate at different wave velocities. Dispersion relationships are very important in discussing the interactions between different types of wave motions (e.g. interactions between sound waves and structural waves). When a wave is non-dispersive, the wave velocity, c, is constant and therefore $\partial \omega / \partial k$ (the gradient of equation 1.2) is also constant. When a wave is dispersive, both the wave velocity, c, and the gradient of the corresponding dispersion relationships are variables. This is illustrated in Figure 1.4. The gradient of the dispersion relationship is termed the group velocity, c

$$c_{\mathbf{g}} = \partial \omega / \partial k, \tag{1.4}$$

and it quantifies the speed at which energy is transported by the dispersive wave. It is the velocity at which an amplitude function which is impressed upon a carrier wave packet (a time-varying wave motion which can be represented as a summation of numerous harmonic waves) travels, and it is of great physical importance. Plane sound waves and compressional waves in solids are typical examples of non-dispersive waves, and flexural waves in solids are typical examples of dispersive waves. If the dispersion relationship of any two types of wave motions intersect, they then have the same frequency, wavenumber, wavelength and wavespeed. This condition (termed 'coincidence') allows for very efficient interactions between the two wave-types, and it will be discussed in some detail in chapters 3 and 7.

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1.3 Introductory multiple, discrete, mass–spring–damper oscillator concepts – a macroscopic viewpoint

When considering the mechanical vibrations of machine elements and structures one generally utilises either the lumped or the distributed parameter approach to study the normal modes of vibration of the system. Engineers are often only concerned with the estimation of the first few natural frequencies of a large variety of structures, and the macroscopic approach with multiple, discrete, mass–spring–damper oscillators is therefore more appropriate (as opposed to the wave approach). When modelling the vibrational characteristics of a structure via the macroscopic approach, the elements that constitute the model include a mass, a spring, a damper and an excitation. The elementary, one-degree-of-freedom, lumped-parameter oscillator model is illustrated in Figure 1.5.

The excitation force provides the system with energy which is subsequently stored by the mass and the spring, and dissipated in the damper. The mass, m, is modelled as a rigid body and it gains or loses kinetic energy. The spring (with a stiffness k_s) is assumed to have a negligible mass, and it possesses elasticity. A spring force exists when there is a relative displacement between its ends, and the work done in compressing or extending the spring is converted into potential energy – i.e. the strain energy is stored in the spring. The spring stiffness, k_s , has units of force per unit deflection. The damper (with a viscous-damping coefficient c_v) has neither mass nor stiffness, and a damping force will be produced when there is relative motion between its ends. The damper is non-conservative because it dissipates energy. Various types of damping models are available, and viscous damping (i.e. the damping force is proportional to velocity) is the most commonly used model. The viscous-damping coefficient, c_v , has units of force per unit velocity. Other damping models include coulomb (or dry-friction) damping,



Fig. 1.5. One-degree-of-freedom, lumped-parameter oscillator.





Fig. 1.6. A simplified, multiple, discrete mass-spring-damper model of a human body standing on a vibrating platform.

hysteretic damping, and velocity-squared damping. Fluid dynamic drag on bodies, for example, approximates to velocity-squared damping (the exact value of the exponent depends on several other variables).

The idealised elements that make up the one-degree-of-freedom system form an elementary macroscopic model of a vibrating system. In general, the models are somewhat more complex and involve multiple, discrete, mass-spring-damper oscillators. In addition, the masses of the various spring components often have to be accounted for (for instance, a coil spring possesses both mass and stiffness). The low frequency vibration characteristics of a large number of continuous systems can be approximated by a finite number of lumped parameters. The human body can be approximated as a linear, lumped-parameter system for the analysis of low frequency (<200 Hz) shock and vibration effects. A simplified multiple, discrete, mass-spring-damper model of a human body standing on a vibrating platform is illustrated in Figure 1.6. The natural frequencies of various parts of the human body can be estimated from such a

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model, and the subsequent effects of external shock and vibration can therefore be analysed.

The concepts of multiple, discrete, mass–spring–damper models can be extended to analyse the vibrations of continuous systems (i.e. systems with an infinite number of degrees of freedom, natural frequencies, and modes of vibration) at higher frequencies by re-modelling the structure in terms of continuous or distributed elements. Mathematically, the problem is usually first set up in terms of the wave equation and subsequently generalised as an eigenvalue problem in terms of modal mass, stiffness and damping. The total response is thus a summation of the modal responses over the frequency range of interest.

It should be noted that the generally accepted convention in most of the literature is the symbol c for both the wave (phase) velocity and the viscous-damping coefficient, and the symbol k for both the wavenumber and the spring stiffness. To avoid this conflicting use of symbols, the symbol c will denote the wave (phase) velocity, the symbol c_v , the viscous-damping coefficient, the symbol k the wavenumber, and the symbol k_s , the spring stiffness.

1.4 Introductory concepts on natural frequencies, modes of vibration, forced vibrations and resonance

Natural frequencies, modes of vibration, forced vibrations and resonance can be described both from an elastic continuum and a macroscopic viewpoint. The existence of natural frequencies and modes of vibration relates to the fact that all real physical systems are bounded in space. A mode of vibration (and the natural frequency associated with it) on a taut, fixed string can be interpreted as being composed of two waves of equal amplitude and wavelength travelling in opposite directions between the two bounded ends. Alternatively, it can be interpreted as being a standing wave, i.e. the string oscillates with a spatially varying amplitude within the confines of a specific stationary waveform. The first interpretation of a mode of vibration relates to the wave model, and the second to the macroscopic model. Both describe the same physical motion and are mathematically equivalent – this will be illustrated in section 1.9.

The concepts discussed above can be illustrated by means of a simple example. Let us consider a piece of string which is stretched and clamped at its ends, as illustrated in Figure 1.7(a). The string is plucked at some arbitrary point and allowed to vibrate freely. At the instant that the string is plucked, a travelling wave is generated in each direction (i.e. towards each clamped end of the string). It is important to recognise that, at this instant, the shape of the travelling wave is not that of a mode of vibration (Figure 1.7b) since a standing wave pattern has yet to be established. The travelling waves move along the string until they meet the clamped ends, at which point they are reflected. After these initial reflections (one from each clamped end) there is a further