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Introduction

In this introductory chapter we focus on the interaction of optical fields with matter because it forms the basis of signal amplification in all optical amplifiers. According to our present understanding, optical fields are made of photons with properties precisely described by the laws of quantum field theory [1]. One consequence of this wave–particle duality is that optical fields can be described, in certain cases, as electromagnetic waves using Maxwell's equations and, in other cases, as a stream of massless particles (photons) such that each photon contains an energy $h\nu$, where h is the Planck constant and ν is the frequency of the optical field. In the case of monochromatic light, it is easy to relate the number of photons contained in an electromagnetic field to its associated energy density. However, this becomes difficult for optical fields that have broad spectral features, unless full statistical features of the signal are known [2]. Fortunately, in most cases that we deal with, such a detailed knowledge of photon statistics is not necessary or even required [3, 4]. Both the linear and the nonlinear optical studies carried out during the last century have shown us convincingly that a theoretical understanding of experimental observations can be gained just by using wave features of the optical fields if they are intense enough to contain more than a few photons [5]. It is this semiclassical approach that we adopt in this book. In cases where such a description is not adequate, one could supplement the wave picture with a quantum description.

In Section 1.1 we introduce Maxwell's equations and the Fourier-transform relations in the temporal and spatial domains used to simplify them. This section also establishes the notation used throughout this book. In Section 1.2 we look at widely used dielectric functions describing dispersive optical response of materials. After discussing dispersion relations in Section 1.3, we show in Section 1.4 that they cannot have an arbitrary form because of the constrained imposed by the causality and enforced by the Kramers–Kronig relations. Section 1.5 considers the propagation of plane waves in a dispersive medium because plane waves play a central role in analyzing various amplification schemes.

1.1 Maxwell's equations

In this section we begin with the time-domain Maxwell's equations and introduce their frequency-domain and momentum-domain forms using the Fourier transforms. These forms are then used to classify different optical materials through the constitutive relations.

1.1.1 Maxwell's equations in the time domain

In a semiclassical approach, Maxwell's equations provide the fundamental basis for the propagation of optical fields through any optical medium [6, 7]. These four equations can be written in an integral form. Two of them relate the electric field vector $\boldsymbol{E}$ and electric flux-density vector $\boldsymbol{D}$ with the magnetic field vector $\boldsymbol{H}$ and the magnetic flux-density vector $\boldsymbol{B}$ using the line and surface integrals calculated over a closed contour l surrounding a surface S_l shown in Figure 1.1:

$$\textbf{Faraday's law of induction:}\ \oint_l \boldsymbol{E}(\boldsymbol{r},t)\cdot d\boldsymbol{l} = -\frac{d}{dt}\int_{S_l} \boldsymbol{B}(\boldsymbol{r},t)\cdot d\boldsymbol{S}, \tag{1.1a}$$

$$\textbf{Ampere's circuital law:}\ \oint_l \boldsymbol{H}(\boldsymbol{r},t)\cdot d\boldsymbol{l} = \frac{d}{dt}\int_{S_l} \boldsymbol{D}(\boldsymbol{r},t)\cdot d\boldsymbol{S} + I(t), \tag{1.1b}$$

where $I(t)$ is the total current flowing across the surface S_l. The pairs $\boldsymbol{E}$, $\boldsymbol{D}$ and $\boldsymbol{H}$, $\boldsymbol{B}$ are not independent even in vacuum and are related to each other by

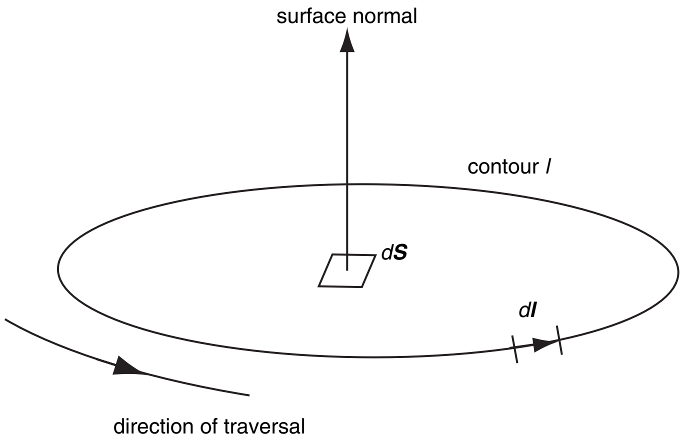

Figure 1.1 Closed contour used for calculating the line and surface integrals appearing in Maxwell's equations.

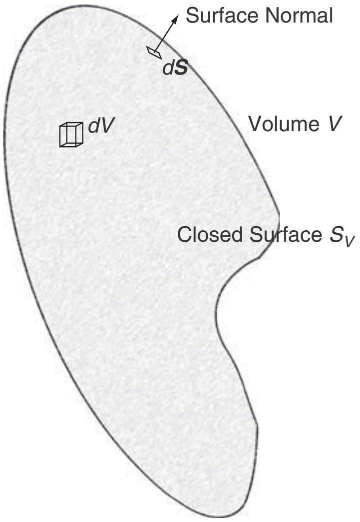

Figure 1.2 Fixed volume used for calculating the surface and volume integrals appearing in Maxwell's equations.

$$\boldsymbol{D} = \varepsilon_0 \boldsymbol{E}, \tag{1.2a}$$

$$\boldsymbol{B} = \mu_0 \boldsymbol{H}, \tag{1.2b}$$

where μ_0 is the permeability and ε_0 is the permittivity in free space.

The other two of Maxwell's equations relate the electric field vector $\boldsymbol{E}$ with the magnetic flux-density vector $\boldsymbol{B}$ using surface integrals calculated over a fixed volume V, bounded by a closed surface S_V as shown in Figure 1.2. Their explicit form is

$$\textbf{Gauss's law:} \quad \oint_{S_V} \boldsymbol{D}(\boldsymbol{r}, t) \cdot d\boldsymbol{S} = q(t), \tag{1.3a}$$

$$\textbf{Gauss's law for magnetism:} \quad \oint_{S_V} \boldsymbol{B}(\boldsymbol{r}, t) \cdot d\boldsymbol{S} = 0, \tag{1.3b}$$

where $q(t)$ is the total electric charge contained in the volume V.

In a continuous optical medium, the four integral equations can be recast in an equivalent differential form useful for theoretical analysis and numerical computations [6, 7]. Application of the *Stokes theorem*[1] to Eqs. (1.1) provides us with

[1] A continuous vector field $\boldsymbol{F}$ defined on a surface S with a boundary l satisfies $\oint_l \boldsymbol{F} \cdot d\boldsymbol{l} = \int_S \nabla \times \boldsymbol{F} \cdot d\boldsymbol{S}$.

Curl equation for electric field: $$\nabla \times \boldsymbol{E}(\boldsymbol{r}, t) = -\frac{\partial \boldsymbol{B}(\boldsymbol{r}, t)}{\partial t}, \tag{1.4a}$$

Curl equation for magnetic field: $$\nabla \times \boldsymbol{H}(\boldsymbol{r}, t) = \frac{\partial \boldsymbol{D}(\boldsymbol{r}, t)}{\partial t} + J(\boldsymbol{r}, t), \tag{1.4b}$$

where $J(\boldsymbol{r}, t)$ is the electric current density. Similarly, application of the *divergence theorem*[2] to Eqs. (1.3) provides us with

Divergence equation for electric field: $$\nabla \cdot \boldsymbol{D}(\boldsymbol{r}, t) = \rho(\boldsymbol{r}, t), \tag{1.5a}$$

Divergence equation for magnetic field: $$\nabla \cdot \boldsymbol{B}(\boldsymbol{r}, t) = 0, \tag{1.5b}$$

where $\rho(\boldsymbol{r}, t)$ is the local charge density.

The current density $J(\boldsymbol{r}, t)$ and the charge density $\rho(\boldsymbol{r}, t)$ are related to each other through Maxwell's equations. This relationship can be established by taking the divergence of Eq. (1.4b) and noting the operator identity $\nabla \cdot (\nabla \times \boldsymbol{F}) \equiv 0$ for any vector field $\boldsymbol{F}$. The result is

$$\nabla \cdot J(\boldsymbol{r}, t) + \frac{\partial \rho(\boldsymbol{r}, t)}{\partial t} = 0. \tag{1.6}$$

This equation is called the charge-continuity equation because it shows that the charge moving out of a differential volume is equal to the rate at which the charge density decreases within that volume. In other words, the continuity equation represents mathematically the principle of charge conservation at each point of space where the electromagnetic field is continuous.

1.1.2 Maxwell's equations in the frequency domain

The differential form of Maxwell's equations can be put into an equivalent format by mapping time variables to the frequency domain [8]. This is done by introducing the Fourier-transform operator $\mathscr{F}_{t+}\{\}(\omega)$ as

$$\begin{aligned}\widetilde{Y}(\ldots, \omega, \ldots) &\triangleq \mathscr{F}_{t+}\{Y(\ldots, t, \ldots)\}(\omega) \\ &= \int_{-\infty}^{\infty} Y(\ldots, t, \ldots) \exp(+j\omega t) dt,\end{aligned} \tag{1.7}$$

where $j = \sqrt{(-1)}$ and ω is the associated frequency-domain variable corresponding to time t; ω can assume any value on the real axis (i.e., $-\infty < \omega < +\infty$). Even though all quantities in the physical world correspond to real variables, their Fourier

[2] A vector field $\boldsymbol{F}$ defined on a volume V with a boundary surface S satisfies $\oint_V \nabla \cdot \boldsymbol{F} dV = \int_S \boldsymbol{F} \cdot d\boldsymbol{S}$.

transforms can result in complex numbers. Nonetheless, for physical and mathematical reasons, the Fourier representation can provide a much simpler description of an underlying problem in certain cases.

We consistently use the notation that a tilde over a time-domain variable Y represents its Fourier transform when the Fourier integral is done with the plus sign in the exponential $\exp(+j\omega t)$. The Fourier operator shows this sign convention by displaying a plus sign just after its integration variable t. It is important to note that by adopting this notation, we also explicitly indicate that the Fourier transform takes the real variable t to its Fourier-space variable ω.

A very useful feature of the Fourier transform is that a function can be readily inverted back to its original temporal form using the inverse Fourier-transform operator $\mathscr{F}^{-1}_{\omega+}\{\}(t)$, defined as

$$\begin{aligned} Y(\ldots,t,\ldots) &\triangleq \mathscr{F}^{-1}_{\omega+}\left\{\widetilde{Y}(\ldots,\omega,\ldots)\right\}(t) \\ &= \frac{1}{(2\pi)^{\dim(\omega)}}\int_{-\infty}^{\infty}\widetilde{Y}(\ldots,\omega,\ldots)\exp(-j\omega t)\,d\omega, \end{aligned} \tag{1.8}$$

where $\dim(\omega)$ is the dimension of the variable ω. Because $t \rightarrow \omega$ is a one-dimensional mapping, we have $\dim(\omega) = 1$ in this instance, but it can assume other positive integer values. For example, the dimension of the Fourier mapping is 3 when we later use spatial Fourier transforms.

To convert Maxwell's equations to the Fourier-transform domain, we need to map the partial differentials in Maxwell's equations to the frequency domain. This can be done by differentiating Eq. (1.8) with respect to t to get

$$\frac{\partial Y(\ldots,t,\ldots)}{\partial t} = \frac{1}{2\pi}\int_{-\infty}^{\infty} -j\omega\widetilde{Y}(\ldots,\omega,\ldots)\exp(-j\omega t)\,d\omega. \tag{1.9}$$

This equation shows that we can establish the operator identity

$$\frac{\partial Y(\ldots,t,\ldots)}{\partial t} \equiv \mathscr{F}^{-1}_{\omega+}\left\{-j\omega\widetilde{Y}(\ldots,\omega,\ldots)\right\}(t), \tag{1.10}$$

resulting in the mapping $\frac{\partial}{\partial t} \rightarrow -j\omega$ from time to frequency domain.

We apply the operator relation (1.10) to the time-domain Maxwell's equations in Eqs. (1.4) and (1.5) to obtain the following frequency-domain Maxwell's equations:

$$\begin{aligned} \nabla\times\widetilde{\boldsymbol{E}}(\boldsymbol{r},\omega) &= j\omega\widetilde{\boldsymbol{B}}(\boldsymbol{r},\omega), \\ \nabla\cdot\widetilde{\boldsymbol{D}}(\boldsymbol{r},\omega) &= \widetilde{\rho}(\boldsymbol{r},\omega), \\ \nabla\times\widetilde{\boldsymbol{H}}(\boldsymbol{r},\omega) &= -j\omega\widetilde{\boldsymbol{D}}(\boldsymbol{r},\omega) + \widetilde{\boldsymbol{J}}(\boldsymbol{r},\omega), \\ \nabla\cdot\widetilde{\boldsymbol{B}}(\boldsymbol{r},\omega) &= 0. \end{aligned} \tag{1.11}$$

1.1.3 Maxwell's equations in the momentum domain

Further simplification of Maxwell's equations is possible by mapping the spatial variable $\boldsymbol{r}$ to its equivalent Fourier-domain variable $\boldsymbol{k}$ (also called the momentum domain or the $\boldsymbol{k}$ space). For physical reasons discussed later, the Fourier transform in the spatial domain is defined with a minus sign in the exponential, i.e.,

$$\begin{aligned} \widehat{Y}(\ldots,\boldsymbol{k},\ldots) &\triangleq \mathscr{F}_{\boldsymbol{r}-}\{Y(\ldots,\boldsymbol{r},\ldots)\}(\boldsymbol{k}) \\ &= \int_{-\infty}^{\infty} Y(\ldots,\boldsymbol{r},\ldots)\exp(-j\boldsymbol{k}\cdot\boldsymbol{r})d\boldsymbol{r}. \end{aligned} \tag{1.12}$$

The Fourier operator shows this sign convention clearly by displaying a minus sign just after its integration variable $\boldsymbol{r}$. The hat symbol over a field variable Y represents its spatial-domain Fourier transform.

We should point out that a second choice exists for the signs used in the temporal ($t \to \omega$) and spatial ($\boldsymbol{r} \to \boldsymbol{k}$) Fourier transforms. The sign convention that we have adopted is often used in physics textbooks. Using Maxwell's equations with this sign convention, one can show that a plane wave of the form $\exp(j\boldsymbol{k}\cdot\boldsymbol{r} - j\omega t)$ moves radially outward from a point source for positive values of $\boldsymbol{r}$ and t [9]. The opposite sign convention, where the temporal variable in Eq. (1.7) carries a negative sign, is widely used in electrical engineering literature.

Similarly to the time-domain Fourier transform, a spatial-domain Fourier transform can be inverted with the following formula:

$$\begin{aligned} Y(\ldots,\boldsymbol{r},\ldots) &\triangleq \mathscr{F}^{-1}_{\boldsymbol{k}-}\left\{\widehat{Y}(\ldots,\boldsymbol{k},\ldots)\right\}(\boldsymbol{r}) \\ &= \frac{1}{(2\pi)^{\dim(\boldsymbol{k})}}\int_{-\infty}^{\infty} \widehat{Y}(\ldots,\boldsymbol{k},\ldots)\exp(+j\boldsymbol{k}\cdot\boldsymbol{r})d\boldsymbol{k}, \end{aligned} \tag{1.13}$$

where $\dim(\boldsymbol{k}) = 3$ because the mapping is done in three-dimensional space. Using a relation similar to that appearing in Eq. (1.10), we can establish the mapping $\nabla \to j\boldsymbol{k}$ from the spatial domain to the $\boldsymbol{k}$ space [10]. Applying this mapping to the differential form of Maxwell's equations (1.4) and (1.5) leads to the $\boldsymbol{k}$-space version of Maxwell's equations:

$$\begin{aligned} j\boldsymbol{k}\times\widehat{\boldsymbol{E}}(\boldsymbol{k},t) &= -\frac{\partial\widehat{\boldsymbol{B}}(\boldsymbol{k},t)}{\partial t}, & j\boldsymbol{k}\cdot\widehat{\boldsymbol{D}}(\boldsymbol{k},t) &= \widehat{\rho}(\boldsymbol{k},t), \\ j\boldsymbol{k}\times\widehat{\boldsymbol{H}}(\boldsymbol{k},t) &= \frac{\partial\widehat{\boldsymbol{D}}(\boldsymbol{k},t)}{\partial t} + \widehat{\boldsymbol{J}}(\boldsymbol{k},t), & j\boldsymbol{k}\cdot\widehat{\boldsymbol{B}}(\boldsymbol{k},t) &= 0. \end{aligned} \tag{1.14}$$

Further simplification occurs if we combine the temporal and spatial mappings to form a $\omega \otimes \boldsymbol{k}$ representation of Maxwell's equations. To achieve this, we define a function that takes us from $t \otimes \boldsymbol{r}$ space to $\omega \otimes \boldsymbol{k}$ space (with a hat over the tilde):

$$\widehat{\widetilde{Y}}(\ldots, \boldsymbol{k}, \ldots, \omega, \ldots) \triangleq \mathscr{F}_{\boldsymbol{r}+,t-}\left\{Y(\ldots, \boldsymbol{r}, \ldots, t, \ldots)\right\}(\boldsymbol{k}, \omega) = \int_{-\infty}^{\infty} Y(\ldots, \boldsymbol{r}, \ldots, t, \ldots) \exp(j\boldsymbol{k}\cdot\boldsymbol{r} - j\omega t)\, d\boldsymbol{r}\, dt. \tag{1.15}$$

Application of this Fourier transform to Maxwell's equations in Eqs. (1.4) and (1.5) gives us their following simple algebraic form:

$$\begin{aligned} \boldsymbol{k} \times \widehat{\widetilde{\boldsymbol{E}}}(\boldsymbol{k}, \omega) &= \omega \widehat{\widetilde{\boldsymbol{B}}}(\boldsymbol{k}, \omega), & \boldsymbol{k} \cdot \widehat{\widetilde{\boldsymbol{D}}}(\boldsymbol{k}, \omega) &= \widehat{\widetilde{\rho}}(\boldsymbol{k}, \omega), \\ j\boldsymbol{k} \times \widehat{\widetilde{\boldsymbol{H}}}(\boldsymbol{k}, \omega) &= -j\omega \widehat{\widetilde{\boldsymbol{D}}}(\boldsymbol{k}, \omega) + \widehat{\widetilde{\boldsymbol{J}}}(\boldsymbol{k}, \omega), & \boldsymbol{k} \cdot \widehat{\widetilde{\boldsymbol{B}}}(\boldsymbol{k}, \omega) &= 0. \end{aligned} \tag{1.16}$$

These vectorial relationships show that, in a linear isotropic medium, the triplets $(\widehat{\widetilde{\boldsymbol{E}}}, \widehat{\widetilde{\boldsymbol{B}}}, \boldsymbol{k})$ and $(\widehat{\widetilde{\boldsymbol{D}}}, \widehat{\widetilde{\boldsymbol{H}}}, \boldsymbol{k})$ form two right-handed coordinate systems shown in Figure 1.3, irrespective of material parameters. However, such a general relationship does not exists for the triplets $(\widehat{\widetilde{\boldsymbol{E}}}, \widehat{\widetilde{\boldsymbol{H}}}, \boldsymbol{k})$ and $(\widehat{\widetilde{\boldsymbol{D}}}, \widehat{\widetilde{\boldsymbol{B}}}, \boldsymbol{k})$. We shall see later that the signs of the permittivity and permeability associated with a medium play an important role in establishing functional relationships between the four field variables.

The preceding Maxwell's equations need to be modified when they are applied to physical materials because of charge movement (conductance), induced polarization, and induced magnetization within the medium. Charge movement in a material occurs because the electric and magnetic fields exert force on charges. Induced polarization within a material medium is a result of the rearrangement of the bound electrons, and it is responsible for the appearance of additional charge density known as the bound charge density. If these bound charges oscillate as a result of an externally applied electromagnetic field, electric dipoles induce a polarization current within the medium. In addition to the electric dipoles, a magnetization current can also be generated if magnetic dipoles are present in a medium.

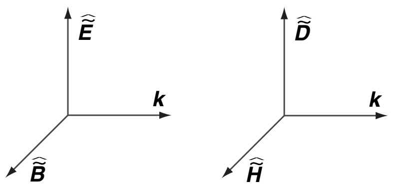

Figure 1.3 An illustration of the orthogonality of the triplets $(\widehat{\widetilde{\boldsymbol{E}}}, \widehat{\widetilde{\boldsymbol{B}}}, \boldsymbol{k})$ and $(\widehat{\widetilde{\boldsymbol{D}}}, \widehat{\widetilde{\boldsymbol{H}}}, \boldsymbol{k})$.

Table 1.1. *Constitutive relations for several types of optical media*

Medium type	Functional dependence	Description
Simple	$\widehat{\widehat{\boldsymbol{D}}} = \varepsilon \widehat{\widehat{\boldsymbol{E}}}$	ε: constant, scalar
	$\widehat{\widehat{\boldsymbol{B}}} = \mu \widehat{\widehat{\boldsymbol{H}}}$	μ: constant, scalar
Dispersive	$\widehat{\widehat{\boldsymbol{D}}} = \widehat{\widehat{\varepsilon}}(\boldsymbol{k}, \omega) \widehat{\widehat{\boldsymbol{E}}}$	$\widehat{\widehat{\varepsilon}}(\boldsymbol{k}, \omega)$: complex function
	$\widehat{\widehat{\boldsymbol{B}}} = \widehat{\widehat{\boldsymbol{\mu}}}(\boldsymbol{k}, \omega) \widehat{\widehat{\boldsymbol{H}}}$	$\widehat{\widehat{\boldsymbol{\mu}}}(\boldsymbol{k}, \omega)$: complex function
Anisotropic	$\widehat{\widehat{\boldsymbol{D}}} = \varepsilon_0 \widehat{\widehat{\boldsymbol{E}}} + \varepsilon_0 \widehat{\widehat{\boldsymbol{\chi}}}_e \cdot \widehat{\widehat{\boldsymbol{E}}}$	$\widehat{\widehat{\boldsymbol{\chi}}}_e$: 3×3 matrix
	$\widehat{\widehat{\boldsymbol{B}}} = \mu_0 \widehat{\widehat{\boldsymbol{H}}} + \mu_0 \widehat{\widehat{\boldsymbol{\chi}}}_m \cdot \widehat{\widehat{\boldsymbol{H}}}$	$\widehat{\widehat{\boldsymbol{\chi}}}_m$: 3×3 matrix
Bi-isotropic	$\widehat{\widehat{\boldsymbol{D}}} = \varepsilon \widehat{\widehat{\boldsymbol{E}}} + \xi \widehat{\widehat{\boldsymbol{H}}}$	ε, ξ: constant, scalars
	$\widehat{\widehat{\boldsymbol{B}}} = \zeta \widehat{\widehat{\boldsymbol{E}}} + \mu \widehat{\widehat{\boldsymbol{H}}}$	ζ, μ: constant, scalars

All of these phenomena are incorporated within Maxwell's equations through the so-called constitutive relations among the four field vectors.

1.1.4 Constitutive relations for different optical media

The most general linear relationship among $\widehat{\widehat{\boldsymbol{E}}}$, $\widehat{\widehat{\boldsymbol{D}}}$, $\widehat{\widehat{\boldsymbol{H}}}$, and $\widehat{\widehat{\boldsymbol{B}}}$ occurs in a bi-anisotropic medium, and it can be written as

$$\widehat{\widehat{\boldsymbol{D}}}(\boldsymbol{k}, \omega) = \widehat{\widehat{\boldsymbol{\varepsilon}}}(\boldsymbol{k}, \omega) \cdot \widehat{\widehat{\boldsymbol{E}}}(\boldsymbol{k}, \omega) + \widehat{\widehat{\boldsymbol{\xi}}}(\boldsymbol{k}, \omega) \cdot \widehat{\widehat{\boldsymbol{H}}}(\boldsymbol{k}, \omega), \tag{1.17a}$$

$$\widehat{\widehat{\boldsymbol{B}}}(\boldsymbol{k}, \omega) = \widehat{\widehat{\boldsymbol{\zeta}}}(\boldsymbol{k}, \omega) \cdot \widehat{\widehat{\boldsymbol{E}}}(\boldsymbol{k}, \omega) + \widehat{\widehat{\boldsymbol{\mu}}}(\boldsymbol{k}, \omega) \cdot \widehat{\widehat{\boldsymbol{H}}}(\boldsymbol{k}, \omega), \tag{1.17b}$$

where $\widehat{\widehat{\boldsymbol{\varepsilon}}}(\boldsymbol{k}, \omega)$, $\widehat{\widehat{\boldsymbol{\xi}}}(\boldsymbol{k}, \omega)$, $\widehat{\widehat{\boldsymbol{\zeta}}}(\boldsymbol{k}, \omega)$, and $\widehat{\widehat{\boldsymbol{\mu}}}(\boldsymbol{k}, \omega)$ are in the form of 3×3 matrices. However, some of those matrices are identically zero or have scalar values depending on the nature of material medium. The constitutive relations for several classes of optical media are classified in Table 1.1, based on the dependence of the four parameters on ω and $\boldsymbol{k}$. A medium is called *frequency dispersive* if these parameters depend explicitly on ω. Similarly, a medium is called *spatially dispersive* if they depend explicitly on $\boldsymbol{k}$.

Frequency dispersion is an inherent feature of any optical medium because no real medium can respond to an electromagnetic field instantaneously. It is possible to show that the frequency dependence of the permittivity of a medium is fundamentally related to the causality of the response of that medium that leads naturally to the Kramers–Kronig relations [11, 12] discussed later. Using these relations one can also show that a frequency-dispersive medium is naturally lossy. Since the commonly used assumption of a lossless dispersive medium is not generally valid, the notion of an optically thick, lossless medium needs to be handled very cautiously. In certain cases, we may consider the frequency dependence of the refractive index but neglect such dependence for the absorption. Generally, these kinds of assumptions lead to the prediction of noncausal behavior (e.g., a signal traveling faster than the of speed of light in vacuum). However, if the frequency dependence of the refractive index is considered only over a finite frequency range, absorption can be neglected over that range without violating causality [13].

We do not consider spatial dispersion in this text because optical wavelengths are much longer than atomic dimensions and inter-atomic spacing. It is interesting to note that spatial dispersion is implicitly related to the magneto-electric response of a medium. In a magneto-electric medium, the presence of electric fields causes the medium to become magnetized, and the presence of magnetic fields makes the medium polarized. It is evident from Eq. (1.16) that the magnetic field is completely determined by the transverse part of the electric field in the $\omega \otimes \boldsymbol{k}$ space. Thus, it is not possible, in principle, to separate the roles of electric and magnetic fields.

Most physical media are neither linear nor isotropic in the sense that their properties depend on both the strength of the local electric field and its direction. Also, there exist optically active media that can rotate the state of polarization of the electric field either clockwise (dextrorotation) or counterclockwise (levorotation). Although we consider the nonlinear nature of an optical medium whenever relevant, unless otherwise stated explicitly we do not cover this type of optically active medium in this text. Furthermore, unless explicitly stated, we assume that the medium is nonmagnetic. In such a medium, the parameters ξ and ζ in Eqs. (1.1) vanish and the magnetic permeability can be replaced with its vacuum value μ_0. With these simplifications, we only need to know the permittivity ε to study wave propagation in a nonmagnetic medium. It is common to introduce the susceptibility χ of the medium by the relation $\varepsilon = \varepsilon_0(1 + \chi)$.

1.2 Permittivity of isotropic materials

In this section we focus on an isotropic, homogeneous medium and ignore the dependence of the permittivity ε on the propagation vector $\boldsymbol{k}$. In the frequency domain, this dielectric function is complex because, as mentioned earlier, it must have a nonzero imaginary part to allow for absorption at certain frequencies that correspond to medium resonances. As we show later, the real and imaginary parts of the dielectric function are related to each other through the Kramers–Kronig relations [11].

1.2.1 Debye-type permittivity and its extensions

In some cases one can model an optical medium as a collection of noninteracting dipoles, each of which responds to the optical field independently. The electromagnetic response of such a medium is given by the Debye dielectric function [14],

$$\tilde{\varepsilon}_{\text{Debye}}(\omega) = \varepsilon_0 \left[\varepsilon_\infty + \frac{\Delta\varepsilon_p}{1 - j\omega\tau_p} \right], \tag{1.18}$$

where ε_∞ is the dielectric constant of the medium in the high-frequency limit and $\Delta\varepsilon_p = \varepsilon_s - \varepsilon_\infty$ is the change in relative permittivity from its static value ε_s owing to the relaxation time τ_p associated with the medium. The parameter ε_∞ represents the deformational electric polarization of positive and negative charge separation due to the presence of an external electric field [15]. The ratio $\varepsilon/\varepsilon_0$ is sometimes referred to as the relative permittivity of the medium.

The permittivity $\varepsilon_{\text{Debye}}(t)$ of the medium in the time domain is obtained by taking the inverse Fourier transform of Eq. (1.18). If we introduce the susceptibility as defined earlier, its time dependence for the Debye dielectric function is exponential and is given by

$$\chi_{\text{Debye}}(t) = \begin{cases} \frac{\Delta\varepsilon_p}{\tau_p} \exp\left(-\frac{t}{\tau_p}\right) & \text{if } t > 0, \\ 0 & \text{otherwise.} \end{cases} \tag{1.19}$$

This equation shows the case of a medium with only one relaxation time. The Debye dielectric function in Eq. (1.18) can be extended readily to a medium with N relaxation times by using

$$\tilde{\varepsilon}_{\text{Debye}}(\omega) = \varepsilon_0 \left[\varepsilon_\infty + \sum_{m=1}^{N} \frac{\Delta\varepsilon_m}{1 - j\omega\tau_m} \right]. \tag{1.20}$$

To account for the asymmetry and broadness of some experimentally observed dielectric functions, a variant of Eq. (1.18) was suggested in Ref. [16]. This variant, known as $\tilde{\varepsilon}_{\text{DHN}}(\omega)$, introduces empirically two parameters, α and β:

$$\tilde{\varepsilon}_{\text{DHN}}(\omega) = \varepsilon_0 \left[\varepsilon_\infty + \frac{\Delta\varepsilon_p}{[1 + (-j\omega\tau_p)^\alpha]^\beta} \right]. \tag{1.21}$$

The value of α is adjusted to match the observed asymmetry in the shape of the permittivity spectrum and β is used to control the broadness of the response. When $\alpha = 1$, this model is known as the Cole–Davidson model [17] and has the form

$$\tilde{\varepsilon}_{\text{CD}}(\omega) = \varepsilon_0 \left[\varepsilon_\infty + \frac{\Delta\varepsilon_p}{(1 - j\omega\tau_p)^\beta} \right], \quad 0 < \beta \leq 1. \tag{1.22}$$