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Antonio Ambrosetti and Giovanni Prodi
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A Primer of Nonlinear Analysis

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Preface

In the last few decades, once linear functional analysis was quite widely and thoroughly estabilished, the interest of scientists in Nonlinear Analysis has been increasing a lot. On the one hand the treatments of various classical problems have been unified; on the other, theories specifically nonlinear, of great significance and applicability, have come out.

This book provides an introduction to basic aspects of Nonlinear Analysis, namely those based on differential calculus in Banach spaces. The matter is expressed in a geometric style, in the sense that the results obtained are often a transposition to infinite dimensions of events which are intuitive in $\mathbb{R}^2$ or $\mathbb{R}^3$. Indeed, this was a primary characteristic of the works of Pincherle, Volterra and Fréchet.

The topics treated can be divided into two main parts and are preceded by a short chapter in which some introductory material is recalled, and also the main notation fixed.

In the first part, differential calculus in Banach spaces is discussed, together with local and global inversion theorems.

The second part deals with bifurcation theory which in spite of its elementary character is, perhaps, one of the most powerful tools used in Nonlinear Analysis. Our attention is here devoted almost entirely to the case of simple eigenvalues, but an accurate analysis of hypotheses is made, in order to include, for example, also the celebrated Hopf theorem.

A specific feature of Nonlinear Analysis is that the theoretical setting is strictly linked to applications, especially those related to differential equations, where the power of nonlinear methods is expressed in a more striking way. Moreover, a relevant fact to be emphasized is that problems that are often considered of formidable difficulty, once they are framed in an appropriate functional setting, may be faced and solved quite easily.

It is, indeed, this aspect, peculiar to Nonlinear Analysis, that has driven us to leave considerable space to applications to differential equations, including various important classical problems such as Bénard Problem, the problem of water waves, the restricted three-body problem and some others. Thus, in addition to more elementary examples and applications that usefully accomplish theoretical results, in separate paragraphs and/or chapters, we deal with those problems which require more care both in formulation and in resolution.

Tools, still of remarkable importance, such as the theory of Leray–Schauder topological degree, or the critical point theory, which would require wider theoretical background and more subtle arguments, are left out in this treatise.

The book in its outlines is self-contained for a reader who, besides infinitesimal calculus, is acquainted with fundamental results of Linear Functional Analysis such as the Hahn–Banach Theorem, the "Closed Graph" Theorem and the Fredholm Alternative Principle. Only some of the problems dealing with partial differential equations require a certain knowledge of Sobolev spaces and therefore, in just a few cases, we refer to results contained in original papers.

This volume is partially based on an earlier booklet, published in Italian by the Scuola Normale Superiore di Pisa in the series "Quaderni". The authors wish to thank the Scuola Normale Superiore for the encouragement.