

Cambridge University Press

978-0-521-48537-1 - Some Asymptotic Problems in the Theory of Partial
Differential Equations

Olga Oleinik

Frontmatter

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In 1993, Professor Oleinik was invited to give a series of lectures about her work in the area of partial differential equations. This book contains those lectures, and more. It is in two parts, the first being devoted to the study of the asymptotic behaviour at infinity of solutions of a class of non-linear second order elliptic equations in unbounded, in particular mathematical physics, such as in the theory of traveling waves, homogenization, boundary layer theory, flame propagation and combustion.

The second part contains the most recent results of the author's research in the theory of homogenization of partial differential equations, and is concerned with questions about partially perforated domains and of solutions with rapidly alternating types of boundary conditions. These new asymptotic problems arise naturally in applications.

Many of the results within have not appeared in book form before, and this volume sheds new light on the subject, raising many new ideas and open problems.

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*Editor: Luigi A. Radicati di Brozolo,
Scuola Normale Superiore, Pisa*

This series of books arises from lectures given under the auspices of the Accademia Nazionale dei Lincei and is sponsored by *Fondazione IBM Italia*.

The lectures, given by international authorities, will range on scientific topics from mathematics and physics through to biology and economics. The books are intended for a broad audience of graduate students and faculty members, and are meant to provide a '*mise au point*' for the subject they deal with.

The symbol of the Accademia, the lynx, is noted for its sharp sightedness; the volumes in the series will be penetrating studies of scientific topics of contemporary interest.

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[More information](#)

Contents

Preface	ix
CHAPTER 1. ASYMPTOTIC PROBLEMS FOR NONLINEAR ELLIPTIC EQUATIONS.	
1.1. Nonlinear elliptic boundary-value problems in unbounded domains and the asymptotic behavior of their solutions.	1
CHAPTER 2. ON THE ASYMPTOTIC BEHAVIOR OF SOLUTIONS OF SOME NONLINEAR ELLIPTIC EQUATIONS IN CYLINDRICAL DOMAINS.	
2.1. On the limits at infinity of solutions of some semilinear second order elliptic equations in cylindrical domains.	13
2.2. On the asymptotics at infinity of solutions in cylindrical domains for a class of elliptic second order equations containing first derivatives.	41
2.3. Asymptotics of solutions of the equation $\Delta u - u ^{p-1}u = 0$, $p > 1$, in cylindrical domains.	72
2.4. Asymptotics of solutions of the equation $\Delta u - e^u = 0$ in cylindrical domains.	91
2.5. On an approach to study asymptotics of solutions in cylindrical domains.	109

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Olga Oleinik
Frontmatter
[More information](#)

viii

Contents

CHAPTER 3. ON THE ASYMPTOTIC BEHAVIOR OF SOLUTIONS
OF NONLINEAR ELLIPTIC EQUATIONS IN A NEIGHBORHOOD OF
A CONIC POINT OF THE BOUNDARY.

- | | |
|---|-----|
| 3.1. Asymptotic behavior of solutions of nonlinear elliptic equations near a conic point of the boundary with the Neumann boundary condition. | 119 |
| 3.2. Asymptotic behavior of solutions of nonlinear elliptic equations near a conic point of the boundary with the Dirichlet boundary condition. | 129 |

CHAPTER 4. ON SOME HOMOGENIZATION PROBLEMS.

- | | |
|---|-----|
| 4.1. A homogenization problem for the Laplace operator in a partially perforated domain with the Dirichlet condition on holes. | 134 |
| 4.2. A homogenization problem for the Laplace operator in a partially perforated domain with the Neumann condition on holes. | 161 |
| 4.3. On homogenization of solutions of the Dirichlet problem in partially perforated domains with nonperiodic structures. | 184 |
| 4.4. On the asymptotics of solutions and eigenvalues of an elliptic problem with rapidly alternating type of boundary conditions. | 191 |

Index	203
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[More information](#)

Preface

This book is based on lectures delivered by the author at the invitation of the Committee of Accademia Nazionale dei Lincei and its President, Professor G. Salvini.

The book consists of two parts and reflects the scientific interests of the author during the last two years. Most of the results presented here are new and are being published for the first time.

The first part of the book is devoted to the study of the asymptotic behavior at infinity of solutions of a class of nonlinear second order elliptic equations in unbounded domains and, in particular, in cylindrical domains. The model equation for this class may be $\Delta u - |u|^{p-1}u = 0$, $p = \text{const} > 1$. Questions of this kind occur in many problems of mathematical physics, in the theory of travelling waves, homogenization, stationary states, boundary layer theory, biology, flame propagation, the probability theory and so on.

The main problems treated in the first part are the study of solutions of the boundary value problems with Dirichlet or Neumann conditions on the lateral part of the boundary of a cylindrical domain and to find the asymptotics of the solutions at infinity.

Specifically, the Gauss equation $\Delta u - e^u = 0$ is considered in one section of the book and the asymptotic properties at infinity of its solutions in cylindrical domains are studied.

Also, the asymptotic behavior of solutions of boundary-value problems in a neighbourhood of a conic point for nonlinear second order equations is investigated. Some new phenomena are found which do not appear in the case of linear equations.

The second part of the book, which contains the most recent results of the author in the theory of homogenization of partial differential equations, is concerned with homogenization in partially perforated domains and homogenization of solutions with rapidly alternating types of boundary conditions.

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Frontmatter
[More information](#)

x

Preface

The problems of homogenization were studied by the author for many years and two of her books on the subject have recently appeared: *Mathematical problems in elasticity and homogenization* (with A.S. Shamaev and G.A. Yosifian, North-Holland, Amsterdam, 1992) and *Homogenization of differential operators and integral functionals* (with V.V. Jikov and S.M. Kozlov, Springer-Verlag, 1994). In these books one can find many basic results on homogenization of partial differential equations as well as the main techniques and methods used in this theory.

The present book raises many interesting open problems some of which are only just beginning to be studied.

The author would like to express her cordial thanks to Professor G. Salvini for his kind invitation, to Professor Gaetano Fichera and Professor G. Kallianpur for their helpful remarks and to Matelda Fichera without whose invaluable help, during the author's stay in Rome, this book could not have been completed.

Olga Oleinik

Rome, 27 October 1993.