

NEW CAMBRIDGE STATISTICAL TABLES

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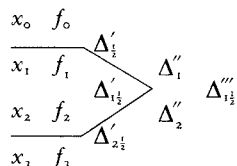
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A NOTE ON INTERPOLATION

Part of the tabulation of a function $f(x)$ at intervals h of x is in the form given in the first two columns of the figure:



where $f_i = f(x_i)$ and $x_{i+1} = x_i + h$. Interpolation of $f(x)$ at values of x other than those tabulated uses the differences in the last three columns, where each entry is the value in the column immediately to the left and below minus the value to the left and above: thus, $\Delta'_{1\frac{1}{2}} = f_2 - f_1$ and $\Delta''_1 = \Delta'_{1\frac{1}{2}} - \Delta'_1$. These are usually written in units of the last place of decimals in $f(x)$. Linear interpolation between x_1 and x_2 approximates $f(x)$ by

$$f_1 + p\Delta'_{1\frac{1}{2}}$$

with $p = (x - x_1)/h$. This simple rule uses only the values within the lines of the figure and is often adequate. Quadratic interpolation between x_1 and x_2 approximates $f(x)$ by

$$f_1 + p\Delta'_{1\frac{1}{2}} - \frac{1}{4}p(1-p)(\Delta''_1 + \Delta''_2).$$

This is generally adequate provided $\Delta'''_{1\frac{1}{2}}$ is less than 60 in units of the last place of decimals in the tabulation. Notice that the quadratic interpolate consists of the addition of an extra term to the linear one, so that a rough assessment of it will indicate whether the linear form is adequate. The maximum possible value of $\frac{1}{4}p(1-p)$ is $-\frac{1}{16}$ when $p = \frac{1}{2}$.

Example. The binomial distribution, $n = 20$, $r = 2$ (Table 1, page 22), interpolation in p , now x .

$\cdot 02$	$\cdot 9929$			
		$- 139$		
$\cdot 03$	$\cdot 9790$		$- 90$	
		$- 229$		3
$\cdot 04$	$\cdot 9561$		$- 87$	
		$- 316$		
$\cdot 05$	$\cdot 9245$			

For $x = 0.034$, $p = (0.034 - 0.03)/0.01 = 0.4$ and the linear interpolate is

$$0.9790 + 0.4 \times (-0.0229) = 0.9698.$$

The additional term for the quadratic interpolate is

$$-0.25 \times 0.4 \times 0.6 \times (-0.0090 - 0.0087) = 0.0011$$

and is not negligible, the quadratic interpolate being 0.9709. This is exact, as is expected since $\Delta'''_{1\frac{1}{2}}$ at 3 is well below 60.

The quadratic method uses f_0 and f_3 (needed for Δ''_1 and Δ''_2) and so fails if either is unavailable, for example at the ends of the range of x or when the interval of tabulation h changes. Modified quadratic forms between x_1 and x_2 are

$$f_1 + p\Delta'_{1\frac{1}{2}} - \frac{1}{2}p(1-p)\Delta''_1 \quad (f_3 \text{ missing}),$$

$$f_1 + p\Delta'_{1\frac{1}{2}} - \frac{1}{2}p(1-p)\Delta''_2 \quad (f_0 \text{ missing}).$$

Occasionally, *harmonic* interpolation is advisable. To do this the argument x is replaced by $1/x$ and then linear (or quadratic) interpolation performed.

Example. The F -distribution, $P = 10$, $v_1 = 1$ (Table 12(a), page 50), interpolation in v_2 , now x .

v_2	$1/v_2$	$F(P)$		
∞	0	2.706		
			42	
120	$1/120$	2.748		1
			43	0
60	$2/120$	2.791		1
			44	
40	$3/120$	2.835		

Notice that the values of v_2 chosen for tabulation are such that the intervals of $1/v_2$ are constant, here $\frac{1}{120}$. The differences show that linear interpolation will be adequate. For $v_2 = 80$, $p = (\frac{1}{80} - \frac{1}{120}) / (\frac{1}{120}) = .5$ and the linear interpolate is $2.748 + 0.5 \times 0.043 = 2.770$ with the possibility of an error of 1 in the last place.

CONSTANTS

$e = 2.71828$	18285	$\log_{10} e = 0.43429$	44819
$\pi = 3.14159$	26536	$\log_e 10 = 2.30258$	50930
$\frac{1}{\sqrt{2\pi}} = 0.39894$	22804	$\log_e \sqrt{2\pi} = 0.91893$	85332

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