NEW CAMBRIDGE STATISTICAL TABLES

D. V. LINDLEY
&
W. F. SCOTT

Second Edition

CAMBRIDGE
UNIVERSITY PRESS

A NOTE ON INTERPOLATION

Part of the tabulation of a function $f(x)$ at intervals h of x is in the form given in the first two columns of the figure:

x_0	f_0			
		$\Delta'_{\frac{1}{2}}$		
x_1	f_1		Δ''_1	
		$\Delta'_{1\frac{1}{2}}$		$\Delta'''_{1\frac{1}{2}}$
x_2	f_2		Δ''_2	
		$\Delta'_{2\frac{1}{2}}$		
x_3	f_3			

where $f_i = f(x_i)$ and $x_{i+1} = x_i + h$. Interpolation of $f(x)$ at values of x other than those tabulated uses the differences in the last three columns, where each entry is the value in the column immediately to the left and below minus the value to the left and above: thus, $\Delta'_{1\frac{1}{2}} = f_2 - f_1$ and $\Delta''_1 = \Delta'_{1\frac{1}{2}} - \Delta'_{\frac{1}{2}}$. These are usually written in units of the last place of decimals in $f(x)$. *Linear* interpolation between x_1 and x_2 approximates $f(x)$ by

$$f_1 + p\Delta'_{1\frac{1}{2}}$$

with $p = (x - x_1)/h$. This simple rule uses only the values within the lines of the figure and is often adequate. *Quadratic* interpolation between x_1 and x_2 approximates $f(x)$ by

$$f_1 + p\Delta'_{1\frac{1}{2}} - \tfrac{1}{4}p(1-p)\,(\Delta''_1 + \Delta''_2).$$

This is generally adequate provided $\Delta'''_{1\frac{1}{2}}$ is less than 60 in units of the last place of decimals in the tabulation. Notice that the quadratic interpolate consists of the addition of an extra term to the linear one, so that a rough assessment of it will indicate whether the linear form is adequate. The maximum possible value of $\frac{1}{4}p(1-p)$ is $-\frac{1}{16}$ when $p = \frac{1}{2}$.

Example. The binomial distribution, $n = 20$, $r = 2$ (Table 1, page 22), interpolation in p, now x.

0·02	0·9929			
		−139		
·03	·9790		−90	
		−229		3
·04	·9561		−87	
		−316		
·05	·9245			

For $x = 0{\cdot}034$, $p = (0{\cdot}034 - 0{\cdot}03)/0{\cdot}01 = 0{\cdot}4$ and the linear interpolate is

$$0{\cdot}9790 + 0{\cdot}4 \times (-0{\cdot}0229) = 0{\cdot}9698.$$

The additional term for the quadratic interpolate is

$$-0{\cdot}25 \times 0{\cdot}4 \times 0{\cdot}6 \times (-0{\cdot}0090 - 0{\cdot}0087) = 0{\cdot}0011$$

and is not negligible, the quadratic interpolate being 0·9709. This is exact, as is expected since $\Delta'''_{1\frac{1}{2}}$ at 3 is well below 60.

The quadratic method uses f_0 and f_3 (needed for Δ''_1 and Δ''_2) and so fails if either is unavailable, for example at the ends of the range of x or when the interval of tabulation h changes. Modified quadratic forms between x_1 and x_2 are

$$f_1 + p\Delta'_{1\frac{1}{2}} - \tfrac{1}{2}p(1-p)\,\Delta''_1 \quad (f_3 \text{ missing}),$$
$$f_1 + p\Delta'_{1\frac{1}{2}} - \tfrac{1}{2}p(1-p)\,\Delta''_2 \quad (f_0 \text{ missing}).$$

Ocassionally, *harmonic* interpolation is advisable. To do this the argument x is replaced by $1/x$ and then linear (or quadratic) interpolation performed.

Example. The F-distribution, $P = 10$, $\nu_1 = 1$ (Table 12(*a*), page 50), interpolation in ν_2, now x.

ν_2	$1/\nu_2$	$F(P)$			
∞	0	2·706			
			42		
120	1/120	2·748		1	
			43		0
60	2/120	2·791		1	
			44		
40	3/120	2·835			

Notice that the values of ν_2 chosen for tabulation are such that the intervals of $1/\nu_2$ are constsnt, here $\frac{1}{120}$. The differences show that linear interpolation will be adequate. For $\nu_2 = 80$, $p = (\frac{1}{80} - \frac{1}{120})/(\frac{1}{120}) = {\cdot}5$ and the linear interpolate is $2{\cdot}748 + 0{\cdot}5 \times 0{\cdot}043 = 2{\cdot}770$ with the possibility of an error of 1 in the last place.

CONSTANTS

$e = 2{\cdot}71828\ 18285$ $\qquad \log_{10} e = 0{\cdot}43429\ 44819$

$\pi = 3{\cdot}14159\ 26536$ $\qquad \log_e 10 = 2{\cdot}30258\ 50930$

$\dfrac{1}{\sqrt{2\pi}} = 0{\cdot}39894\ 22804$ $\qquad \log_e \sqrt{2\pi} = 0{\cdot}91893\ 85332$

CAMBRIDGE
UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom

Cambridge University Press is part of the University of Cambridge.
It furthers the University's mission by disseminating knowledge in the pursuit of education, learning and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9780521484855

First published 1984
Eighth printing 1994
Second edition 1995
20th printing 2015

Printed in the United Kingdom by TJ International Ltd, Pastow, Cornwall

A catalogue record for this publication is available from the British Library

ISBN 978-0-521-48485-5 Paperback