

Cambridge University Press

0521483700 - Prolegomena to a Middlebrow Arithmetic of Curves of Genus 2

J. W. S. Cassels and E. V. Flynn

Table of Contents

[More information](#)

# Contents

*Chapters are divided into sections. Some topics occupy several sections. We give a name to a section and list it here only when it starts a new topic.*

**Foreword** . . . . . ix

## Background and conventions

0. Introduction . . . . . xi  
 1. Algebraically closed ground field . . . . . xi  
 2. Perfect ground field . . . . . xii  
 Appendix. Finite-dimensional Galois modules . . . . . xiii

## Chapter 1. Curves of genus 2

1. Canonical form . . . . . 1  
 2. Group law . . . . . 3  
 3. The form is canonical . . . . . 4  
 4. Plane quartics . . . . . 4

## Chapter 2. Construction of the jacobian

0. Introduction . . . . . 6  
 1. Construction of a basis . . . . . 6  
 3. Local behaviour . . . . . 9  
 4. Formal group law . . . . . 10  
 Appendix I. Blowing up and blowing down . . . . . 12  
 Appendix II. Change of coordinates on  $\mathcal{C}$  . . . . . 13

## Chapter 3. The Kummer surface

0. Introduction . . . . . 17  
 1. Construction of the surface. Nodes . . . . . 18  
 2. Addition of 2-division points . . . . . 20  
 3. Commutativity properties . . . . . 22  
 4. The biquadratic forms . . . . . 23  
 6. A numerical example . . . . . 24  
 7. The tropes . . . . . 25  
 8. The jacobian as double cover . . . . . 26  
 9. The group law on the jacobian . . . . . 27  
 10. Recovery of the curve . . . . . 29

**Chapter 4. The dual of the Kummer**

- 0. Introduction . . . . . 31
- 1. Description of  $\text{Pic}^3$ . . . . . 31
- 3.  $\mathcal{K}^*$  is the projective dual of  $\mathcal{K}$  . . . . . 34
- 4. Tangency . . . . . 36
- 5. Explicit dualities . . . . . 37
- Appendix. Rational divisor classes with no rational divisor . . . . . 39

**Chapter 5. Weddle’s surface**

- 0. Introduction . . . . . 40
- 1. Symmetroid and Jacobian . . . . . 41
- 2. A special case . . . . . 42
- 4. Duality . . . . . 44

**Chapter 6.  $\mathfrak{S}/2\mathfrak{S}$**

- 0. Introduction . . . . . 47
- 1. The homomorphism . . . . . 48
- 2. The kernel . . . . . 50
- 5. When is  $\mathfrak{W} \subset 2\mathfrak{S}$ ? . . . . . 55
- 6. The Kummer viewpoint . . . . . 57
- 8. The norm of  $\rho$  . . . . . 59
- 9. A pathology . . . . . 61
- 10. The quintic case . . . . . 61

**Chapter 7. The jacobian over local fields. Formal groups**

- 0. Introduction . . . . . 63
- 1. Computing the formal group . . . . . 63
- 2. General properties of formal groups . . . . . 67
- 3. The reduction map . . . . . 69
- 4. Torsion in the kernel of reduction . . . . . 70
- 5. The order of  $\mathfrak{S}/2\mathfrak{S}$  when  $k$  is a finite extension of  $\mathbb{Q}_p$ ,  $p \neq \infty$  . . . . . 70
- 6. The order of  $\mathfrak{S}/2\mathfrak{S}$  when  $k = \mathbb{R}$  or  $\mathbb{C}$  . . . . . 73

**Chapter 8. Torsion**

- 0. Introduction . . . . . 75
- 1. Computing the group law . . . . . 75
- 2. Computing the torsion of a given jacobian . . . . . 78
- 3. Searching for large rational torsion . . . . . 82

**Chapter 9. The isogeny. Theory**

- 0. Introduction . . . . . 88
- 4. The Kummer formulation . . . . . 91
- 5. Statement of results . . . . . 92
- 6. Motivation . . . . . 93
- 7. The isogeny . . . . . 95
- 8. Generation by radicals . . . . . 96
- 9. Consequences for Mordell-Weil . . . . . 97
- 10. Isogeny for the curve . . . . . 98

**Chapter 10. The isogeny. Applications**

- 0. Introduction . . . . . 101
- 1. The isogeny on the jacobian variety . . . . . 101
- 2. An injection on  $\mathfrak{E}/\phi(\mathfrak{E})$  . . . . . 104
- 3. Homogeneous spaces  $J_{d_1, d_2}^\phi$  and norm spaces  $L_{d_1, d_2}^\phi$  . . . . . 109
- 4. Computing  $\#\widehat{\mathfrak{E}}/\phi(\mathfrak{E}) \cdot \#\mathfrak{E}/\hat{\phi}(\widehat{\mathfrak{E}})$  when  $k = \mathbb{Q}_p$  . . . . . 110

**Chapter 11. Computing the Mordell-Weil group**

- 0. Introduction . . . . . 113
- 1. The Weak Mordell-Weil Theorem . . . . . 113
- 2. Performing 2-descent without using homogeneous spaces . . . . . 116
- 3. A worked example of complete 2-descent . . . . . 121
- 4. A worked example of descent via isogeny . . . . . 125
- 5. Large rank . . . . . 130

**Chapter 12. Heights**

- 0. Introduction . . . . . 133
- 1. A height function on  $\mathfrak{E}$  . . . . . 133
- 2. The Mordell-Weil Theorem . . . . . 137
- 3. A computational improvement . . . . . 140

**Chapter 13. Rational points. Chabauty's Theorem**

- 0. Introduction . . . . . 143
- 1. Chabauty's Theorem . . . . . 143
- 2. A worked example . . . . . 148

**Chapter 14. Reducible jacobians**

- 0. Introduction . . . . . 154
- 1. The straightforward case . . . . . 154
- 2. An awful warning . . . . . 156
- 3. The reverse process . . . . . 157
- 4. Trying to show that a given jacobian is simple . . . . . 157

Cambridge University Press

0521483700 - Prolegomena to a Middlebrow Arithmetic of Curves of Genus 2

J. W. S. Cassels and E. V. Flynn

Table of Contents

[More information](#)

viii

*Contents***Chapter 15. The endomorphism ring**

- 0. Introduction . . . . . 160
- 1. A few examples . . . . . 160
- 2. Complex and real multiplication . . . . . 163

**Chapter 16. The desingularized Kummer**

- 0. Introduction . . . . . 165
- 1. The surface . . . . . 165
- 2. Elementary properties . . . . . 167
- 3. Equivalence with  $\mathcal{K}$  . . . . . 168
- 4. Equivalence with  $\mathcal{K}^*$  . . . . . 170
- 5. Comparison of maps . . . . . 172
- 6. Further possible developments . . . . . 172

**Chapter 17. A neoclassical approach**

- 0. Introduction . . . . . 174
- 1. Proof of theorem . . . . . 176
- 2. The Kummer surface . . . . . 179
- 3. A special case . . . . . 181
- 4. The construction . . . . . 183

**Chapter 18. Zukunftsmusik**

- 0. Introduction . . . . . 186
- 1. Framework . . . . . 186
- 2. Principal homogeneous spaces . . . . . 186
- 3. Duality . . . . . 186
- 4. Canonical height . . . . . 186
- 5. Second descents . . . . . 187
- 6. Other genera . . . . . 187
- 7. A challenge of Serre's . . . . . 187

**Appendix I. MAPLE programs . . . . . 190****Appendix II. Files available by anonymous ftp . . . . . 204****Bibliography . . . . . 207****Index rerum et personarum . . . . . 219**