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Lectures on Vector Bundles

J. Le Potier

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Preface

This work consists of two courses treating the moduli spaces of vector bundles. The first part is an elementary DEA course which we have given at the Université Denis Diderot (Paris VII) in the Spring of 1991: the subject tackled is the classification of vector bundles on algebraic curves, the construction of the moduli spaces of stable bundles and the first properties of these moduli spaces. In particular, one will find the smoothness criteria and the existence conditions of stable bundles of given rank and degree. It consists of some reminders of facts from algebraic geometry and algebraic topology, the construction, in the case of curves, of the Hilbert-Grothendieck scheme of coherent quotient sheaves of a given bundle, and to treat this we also give a succinct description of Mumford's geometric invariant theory.

The second part is a more advanced course given at the school "Vector Bundles on Surfaces" organized by EUROPROJ (European projective geometry network) and CIMI (Centre International pour les Mathématiques et l'Informatique) at Nice Sophia-Antipolis (14–18 June 1993). The subject we treated here centred on the structure of the moduli space of semi-stable sheaves on the projective plane. In particular, we establish the existence conditions of semi-stable sheaves of given rank and Chern classes and deal with the question of irreducibility and describe the Picard group of the moduli spaces. The construction of the moduli space of semi-stable sheaves is sketched in the more general situation of projective algebraic surfaces: it is fairly similar to the construction we shall see in the first part for the case of curves once we have established the fact that the family of semi-stable sheaves with fixed Hilbert polynomial is bounded; in fact, it generalizes without much dificulty to higher dimensional varieties. The fact that this family is bounded rests on the theorems dealing with restriction of semi-stable sheaves to curves which lead to Bogomolov's Theorem: the statement we shall show here is due to Flenner [20]. This course assumes knowledge of Chern classes of vector bundles and of coherent algebraic sheaves and all the fundamental results of algebraic geometry, notably Serre's Theorems A and B, the Finiteness Theorem and the Riemann-Roch formula, some of which is dealt with in the first part and for the rest the reader could consult, for example, Hartshorne's book [31].

I would like to thank Christoph Sorger who delivered and revised the first version of chapter 6 which deals with Mumford's geometric invariant theory. The enormous work of translation was carried out by Antony Maciocia. I would like to express my satisfaction and my gratitude to him for the result.

Paris, 22nd October 1995.
J. Le Potier

By algebraic variety we mean a separated scheme of finite type over $\mathbb{C}$ and by points we shall always mean closed points.