The movement of oceanic water has important consequences for a variety of applications, such as climate change, sealevel change, biological productivity, weather forecasting, and many others. This book addresses the problem of inferring the state of the ocean circulation, understanding it dynamically, and even forecasting it through a quantitative combination of theory and observation. It focuses on so-called inverse methods and related methods of statistical inference. Both time-independent and time-dependent problems are considered, including Gauss-Markov estimation, sequential estimators, and adjoint/Pontryagin principle methods.

This book is intended for use as a graduate-level text for students of oceanography and other related fields. It will also be of interest to working physical oceanographers.

THE OCEAN CIRCULATION INVERSE PROBLEM

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# Preface

Physical oceanography is a branch of fluid dynamics and is a part of classical physics. As such, the oceanographer's job is to produce quantitative descriptions and explanations of the behavior of the fluid ocean. The movement of oceanic water has consequences for a bewildering variety of applications–climate change; biological productivity; sealevel change; weather forecasting; fisheries prosperity; the chemical history of the earth; the dynamics of the earth–moon system; the movement of pollutants; and so forth. Understanding of the fluid circulation and the properties it carries is of great and growing importance.

Fluid flows are diverse and often very complicated. For this reason, most understanding of particular situations has resulted from an intimate partnership of theory with observation and with laboratory experimentation. But as compared to fluid dynamics as practiced in its innumerable applications–meteorology, aerodynamics, hydraulics, heat transfer, etc.–the problem of observing the ocean is particularly difficult, rivaled perhaps only by the observational problems of inferring the fluid properties of the earth's interior, or of other planets and of stellar interiors. The ocean is very large, turbulent, and inaccessible to electromagnetic radiation. Armed mainly with slow-moving, expensive ships, and instruments which have to work in a corrosive, high-pressure environment, oceanographers have over the years built up in somewhat painful fashion a picture of how the ocean operates. But the picture is known to be badly distorted by the very limited observational base, leading for example, to the need to assume that the large-scale fluid flow is steady with time, so that measurements obtained over many years could be combined in the inferential process. Much of the description available is only qualitative.

But physical oceanography is changing rapidly. New, global-scale, high-resolution general circulation models have been appearing which demand myriad more observations to initialize and to test. New technologies, which are capable of producing those observations, are now at hand. But observations will nonetheless remain expensive and scarce relative to the need. The question of what is best measured, and how to best exploit what data there are, will become even more urgent. Furthermore, as the question of the influence of the oceanic circulation on climate has come to the fore, governments have begun to insist upon the need for quantitative estimates of the future state of the ocean–its fluxes of heat and nutrients, sealevel rise, etc. The stakes are much higher than in the past.

This book is directed at a discussion of the problems of data analysis in the presence of dynamical models with a focus on the general circulation of the ocean. It is an amalgam of ocean dynamics, rudimentary statistics, and bits of linear algebra. The result should be practical for students, working oceanographers, and others faced with the problem of extracting quantitatively useful results from their hard-won observations. As oceanography has emerged as an important component of global change problems, it has attracted specialists from outside the field, for example, those entering from backgrounds in satellite altimeters, or acoustics, or meteorology. I have tried to provide some rough guidance between the Scylla of pure geophysical fluid dynamics and the Charybdis of the overwhelming observational detail already known, in an effort to help newcomers steer a safe and useful course.

But some of the theory of data analysis is interesting in its own right, for example, the Hamiltonian structure of the Riccati equation used for computing the error in sequential estimation. This theory is generally unknown to theoretically inclined oceanographers, and I hope that some of them will find it sufficiently interesting to push the frontiers of our understanding. Following the well-known book by Whittaker and Robinson (1944), we might call the subject "the calculus of observations."

In contrast to the recent work of Bennett (1992), the treatment is almost wholly devoted to finite-dimensional problems and methods. Experience suggests that finite-dimensional techniques are more readily accessible to students and working scientists. These techniques are sufficiently powerful and flexible in their own right that there are few, if any, practical problems which cannot be adequately handled in this way.

I have tried to be careful about terminology, although not claiming to be wholly consistent. The subject is difficult and confusing enough without the added burden of the use of terms whose meaning is unclear or ambiguous. So, for example, the term *barotropic*, which derives originally from the expression *autobarotropic* fluid (meaning one whose pressure and density surfaces coincide), has been so debased by sloppy usage that it can no longer be used without specific definition in context. Similarly, *Sverdrup-relation* is in grave danger of losing its precise meaning, to describe the relationship between the windstress-curl and the integrated meridional flow, and is slipping into the vague implication of validity of the linear vorticity balance. Such loss of precision diminishes the scientist's ability to express ideas in an efficient, clear manner.

## Acknowledgments

I have received highly useful comments and corrections from numerous students who put up with early mistake- and typographical error–ridden versions of this book, used as class notes over several years. A number of colleagues read all or part of the book in great detail, and I am especially indebted to Detlef Stammer, Jochem Marotzke, Joseph Pedlosky, Jürgen Willebrand, Harry Bryden, and Andrew Bennett for saving me from many blunders and numerous obscurities. They are not to blame for those that remain. Particular thanks are due to Dimitris Menemenlis for a virtual line-by-line check of the whole book. Gordon (Bud) Brown patiently worked over the endless diagrams, and the book would not exist without the tireless word-processing skills of William McMechan, who made the initial conversion to TEX and then dealt with endless revisions–all without ever seeing me face-to-face–a working relationship possible only in the era of overnight mail service, fax machines, and e-mail. The sabbatical hospitality of the Geophysical Fluid Dynamics Laboratory, Princeton, and of the Group de Recherche en Géodésie Spatiale, Toulouse, France, contributed considerably to the completion of the manuscript.

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# Notation

| | |
|---|---|
| $\mathbf{A}$ | $N \times N$ model state transition matrix, or $p \times N$ model constraint matrix. |
| $A, A_v \dots$ | Coefficients of eddy diffusion (eddy coefficients). |
| $\mathbf{B}$ | $N \times p$ distribution matrix for known model controls. |
| $\mathbf{C} \equiv <(\tilde{\mathbf{x}} - <\tilde{\mathbf{x}}>)(\tilde{\mathbf{x}} - <\tilde{\mathbf{x}}>)^T>$ | $N \times N$ solution covariance matrix. Sometimes subscripted. |
| $\mathbf{E}$ | $M \times N$ observation or design matrix. |
| $\mathbf{G}$ | Green's function; also matrix of orthogonal vectors (Ch. 3). |
| $J, J'$ | Objective functions. |
| $M$ | Number of observations. |
| $N$ | Size of the state vector. |
| $\mathbf{P} \equiv <(\tilde{\mathbf{x}} - \mathbf{x})(\tilde{\mathbf{x}} - \mathbf{x})^T>$ | $N \times N$ solution uncertainty matrix. |
| $\mathbf{P}_{nn}$ | $M \times M$ residual (noise) uncertainty matrix. |
| $\mathbf{Q} \equiv <(\tilde{\mathbf{u}} - \mathbf{u})(\tilde{\mathbf{u}} - \mathbf{u})^T>$ | Model error second moment matrix. |
| $\mathbf{R}_{xy} \equiv <(\tilde{\mathbf{x}} - \mathbf{x})(\tilde{\mathbf{y}} - \mathbf{y})^T>$ | Second moment matrix. |
| $\mathbf{S}$ | Salinity distribution in a hydrographic section. |
| $T$ | As superscript, the matrix transpose. |
| $\mathbf{T}$ | Mass transport matrix in a hydrographic section (Ch. 4). |
| $\mathbf{T}_u$ | $M \times M$ data resolution matrix. |
| $\mathbf{T}_v$ | $N \times N$ solution resolution matrix. |

| | |
|---|---|
| $\mathbf{U}$ | $M \times M$ matrix of $\mathbf{u}_i$ singular vectors. |
| $\mathbf{U}_K$ | $M \times K$ matrix of first $K$ $\mathbf{u}_i$ singular vectors. |
| $\mathbf{V}$ | $N \times N$ matrix of $\mathbf{v}_i$ singular vectors. |
| $\mathbf{V}_K$ | $N \times K$ matrix of first $K$ $\mathbf{v}_i$ singular vectors. |
| $c, b$ | Reference-level velocities (flows at a reference level used to computer $u_R, v_R$) so absolute geostrophic velocity is $u_R + c$, $v_R + b$. |
| $\mathbf{n}$ | $M \times 1$ noise or residual vector. |
| $\mathbf{n}(t)$ | $M \times 1$ noise or residual vector at time $t$, in which case $\mathbf{n}$ is $M \cdot$ (number of time steps) $\times 1$. |
| $p$ | Pressure field. |
| $t$ | Time variable, either continuous or discrete. |
| $\mathbf{v} = (u, v, w)$ | Components of fluid velocity in $(x, y, z)$ directions in Cartesian coordinates. |
| $u_R, v_R$ | Geostrophic relative velocities (computed relative to a reference level). |
| $\mathbf{u}(t)$ | Control vector (Ch. 6) in time-evolving models. |
| $vec(\ )$ | Vector operator, converting a matrix to a vector by column stacking. |
| $(x, y, z)$ | Cartesian coordinates. |
| $\mathbf{x}$ | $N \times 1$ state vector. |
| $\mathbf{x}(t)$ | $N \times 1$ state vector at time $t$, in which case $\mathbf{x}$ is $N \cdot$ (number of time steps) $\times 1$. |
| $\boldsymbol{\Gamma}$ | Control matrix in model evolution equation. |
| $\boldsymbol{\Lambda}$ | $M \times N$ diagonal matrix of singular values $\lambda_i$. |
| $\boldsymbol{\Lambda}_K$ | $K \times K$ reduced version of $\boldsymbol{\Lambda}$. |
| $\alpha^2$ | Trade-off parameter. |
| $\theta$ | Temperature, or potential temperature. |
| $\lambda$ | Longitude. |
| $\lambda_i$ | Singular values. |
| $\boldsymbol{\mu}$ | Vector of Lagrangian multipliers. |
| $\rho$ | Fluid density. |
| $\sigma$ | Standard deviation, sometimes with subscripts. |
| $\boldsymbol{\tau} = (\tau_x, \tau_y)$ | Wind-stress vector. |
| $\phi$ | Latitude. |
| $\chi_v^2$ | "Chi-square" probability variate, with $v$-degrees of freedom. |