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An Algebraic Introduction to Complex Projective Geometry

1. Commutative algebra

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Introduction

Commutative algebra is the theory of commutative rings and their modules. Although it is an interesting theory in itself, it is generally seen as a tool for geometry and number theory. This is my point of view. In this book I try to organize and present a cohesive set of methods in commutative algebra, for use in geometry. As indicated in the title, I maintain throughout the text a view towards complex projective geometry.

In many recent algebraic geometry books, commutative algebra is often treated as a poor relation. One occasionally refers to it, but only reluctantly. It also suffers from having attracted too much attention thirty years ago. One or several texts are usually recommended: the "Introduction to Commutative Algebra" by Atiyah and Macdonald is a classic for beginners and Matsumura's "Commutative rings" is better adapted for more advanced students. Both these books are excellent and most readers think that there is no need for any other. Today's students seldom consult Bourbaki's books on commutative algebra or the algebra part in the E.G.A. of Grothendieck and Dieudonné.

With this book, I want to prepare systematically the ground for an algebraic introduction to complex projective geometry. It is intended to be read by undergradute students who have had a course in linear and multilinear algebra and know a bit about groups. They may have heard about commutative rings before, but apart from $\mathbb{Z}$ and polynomial rings in one variable with coefficients in $\mathbb{R}$ or $\mathbb{C}$, they have essentially worked with fields. I had to develop quite a lot of language new to them, but I have been careful to articulate all chapters around at least one important theorem. Furthermore I have tried to stimulate readers, whenever their attention may be drifting away, by presenting an example, or by giving them an exercise to solve.

In the first eight chapters, the general theory of rings and modules is developed. I put as much emphasis on modules as on rings; in modern algebraic geometry, sheaves and bundles play as important a role as varieties. I had to decide on the amount of homological algebra that should be included and on the form it should take. This is difficult since the border between commutative and homological algebras is not well-defined. I made several conventional choices. For example, I did not elaborate immediately on the homological nature of length. But quite early on, when studying dualizing modules on Artinian rings in chapter six, I used non-elementary homological methods. In chapter seven, I have been particularly careful on rings and modules of fractions, hoping to prepare readers for working with sheaves.

I wanted this book to be self-contained. Consequently, the basic Galois theory had to be included. I slipped it in at the end of this first part, in chapter nine, just after the study of integral ring extensions.

Now, our favourite ring is $\mathbb{C}[X_1, \ldots, X_n]$, the polynomial ring in several variables with coefficients in the field of complex numbers. We can derive many rings from this one by natural algebraic procedures, even though our purposes are geometric. Quotient rings of $\mathbb{C}[X_1, \ldots, X_n]$, in other words, finitely generated $\mathbb{C}$-algebras, and their fraction rings are the basic objects from chapter ten to chapter thirteen. Noether's normalisation lemma and Hilbert's Nullstellensatz, two splendid theorems of commutative algebra, concern these rings and are at the heart of algebraic geometry. With these results in view, I discuss the notion of dimension and move heartily towards geometry by introducing affine complex schemes and their morphisms. I can then present and prove two other important geometric results, a local version of Zariski's main theorem and Chevalley's semi-continuity theorem.

From chapter fourteen on, I have tried to provide a solid background for modern intersection theory by presenting a detailed study of Weil and Cartier divisors.

In order to keep this book short, I have had to make many painful choices. Several of the chapters that I have deleted from this text will appear in a second book, intended for graduate students, and devoted to homological algebra and complex projective geometry.

I have been careless with the historical background. But I have been careful in developing the material slowly, at least initially, though it does become progressively more difficult as the text proceeds. When necessary, examples and exercises are included within chapters. I allow myself to refer to some of those. All chapters are followed by a series of exercises. Many are easy and a few are more intricate; readers will have to make their own evaluation!

I would like to thank the many students, undergraduates or graduates, whom I taught, or with whom I discussed algebra, at the universities of Strasbourg, Oslo and Paris VI. I have tried to attract each of them to algebra and algebraic geometry. Special thanks are due to Benedicte Basili who wrote a first set of notes from my graduate course in algebraic geometry and introduced me to LaTeX.

My wife Vivi has been tremendously patient while I was writing this first book. I hope very much that some readers will think that she did well in being so.