

Contents

<i>Preface</i>	ix
<i>Notation</i>	xiii
1 Introduction	1
1.1 Vector (linear) spaces	2
1.2 The scalar product	4
1.3 Complex numbers	6
1.4 Quaternions	7
1.5 The cross product	10
1.6 The outer product	11
1.7 Notes	17
1.8 Exercises	18
2 Geometric algebra in two and three dimensions	20
2.1 A new product for vectors	21
2.2 An outline of geometric algebra	23
2.3 Geometric algebra of the plane	24
2.4 The geometric algebra of space	29
2.5 Conventions	38
2.6 Reflections	40
2.7 Rotations	43
2.8 Notes	51
2.9 Exercises	52
3 Classical mechanics	54
3.1 Elementary principles	55
3.2 Two-body central force interactions	59
3.3 Celestial mechanics and perturbations	64

CONTENTS

3.4	Rotating systems and rigid-body motion	69
3.5	Notes	81
3.6	Exercises	82
4	Foundations of geometric algebra	84
4.1	Axiomatic development	85
4.2	Rotations and reflections	97
4.3	Bases, frames and components	100
4.4	Linear algebra	103
4.5	Tensors and components	115
4.6	Notes	122
4.7	Exercises	124
5	Relativity and spacetime	126
5.1	An algebra for spacetime	127
5.2	Observers, trajectories and frames	131
5.3	Lorentz transformations	138
5.4	The Lorentz group	143
5.5	Spacetime dynamics	150
5.6	Notes	163
5.7	Exercises	164
6	Geometric calculus	167
6.1	The vector derivative	168
6.2	Curvilinear coordinates	173
6.3	Analytic functions	178
6.4	Directed integration theory	183
6.5	Embedded surfaces and vector manifolds	202
6.6	Elasticity	220
6.7	Notes	224
6.8	Exercises	225
7	Classical electrodynamics	228
7.1	Maxwell's equations	229
7.2	Integral and conservation theorems	235
7.3	The electromagnetic field of a point charge	241
7.4	Electromagnetic waves	251
7.5	Scattering and diffraction	258
7.6	Scattering	261
7.7	Notes	264
7.8	Exercises	265

CONTENTS

8	Quantum theory and spinors	267
8.1	Non-relativistic quantum spin	267
8.2	Relativistic quantum states	278
8.3	The Dirac equation	281
8.4	Central potentials	288
8.5	Scattering theory	297
8.6	Notes	305
8.7	Exercises	307
9	Multiparticle states and quantum entanglement	309
9.1	Many-body quantum theory	310
9.2	Multiparticle spacetime algebra	315
9.3	Systems of two particles	319
9.4	Relativistic states and operators	325
9.5	Two-spinor calculus	332
9.6	Notes	337
9.7	Exercises	337
10	Geometry	340
10.1	Projective geometry	341
10.2	Conformal geometry	351
10.3	Conformal transformations	355
10.4	Geometric primitives in conformal space	360
10.5	Intersection and reflection in conformal space	365
10.6	Non-Euclidean geometry	370
10.7	Spacetime conformal geometry	383
10.8	Notes	390
10.9	Exercises	391
11	Further topics in calculus and group theory	394
11.1	Multivector calculus	394
11.2	Grassmann calculus	399
11.3	Lie groups	401
11.4	Complex structures and unitary groups	408
11.5	The general linear group	412
11.6	Notes	416
11.7	Exercises	417
12	Lagrangian and Hamiltonian techniques	420
12.1	The Euler–Lagrange equations	421
12.2	Classical models for spin-1/2 particles	427
12.3	Hamiltonian techniques	432

CONTENTS

12.4	Lagrangian field theory	439
12.5	Notes	444
12.6	Exercises	445
13	Symmetry and gauge theory	448
13.1	Conservation laws in field theory	449
13.2	Electromagnetism	453
13.3	Dirac theory	457
13.4	Gauge principles for gravitation	466
13.5	The gravitational field equations	474
13.6	The structure of the Riemann tensor	490
13.7	Notes	495
13.8	Exercises	495
14	Gravitation	497
14.1	Solving the field equations	498
14.2	Spherically-symmetric systems	500
14.3	Schwarzschild black holes	510
14.4	Quantum mechanics in a black hole background	524
14.5	Cosmology	535
14.6	Cylindrical systems	543
14.7	Axially-symmetric systems	551
14.8	Notes	564
14.9	Exercises	565
	<i>Bibliography</i>	568
	<i>Index</i>	575