*Part I*

# *General theory*

# 1

# *Introduction*



## 1.0 Introduction

Optics is the study of light whereas visual optics is the study of the optical properties of the eye and sight. Ancient civilizations such as those of Greece were familiar with some of the properties of light, for example the laws of reflection. However, the Greeks misunderstood the nature of sight and the optical principles of the eye. They believed that light was emitted by the eye and only produced a visual response when the emitted rays struck an object. Many centuries passed before it was realized that light passes from the object to the eye and not from the eye to the object.

We will see later in this book, when we come to look at the optics of the eye, that the ability to sense the visual word around us is limited by the optical properties of the eye and its defects. For example before the advent of optical instruments, the smallest creature that could be seen with the unaided eye was about 0.05 mm in length and the mountains of the moon were unknown. Of particular frustration must have been the deterioration of eyesight with age. For example, as we age, the closest point of clear sight recedes, making it more and more difficult to do some things that we enjoy or need to do, such as reading and fine craft work. The discovery or invention of optical instruments enabled these restrictions to be overcome and allowed mankind to discover a world that was much more complex than ever envisaged, from the discovery of micro flora and fauna to countless galaxies far out in space.

The development of visual optical instruments took place over many centuries and the earliest instruments were developed without any knowledge of how they worked. The first visual optical instrument was probably the spectacle lens which appeared in Europe about 1200 A.D., although it is possible that spherical balls or beads of glass had been used as magnifying lenses well before that. The telescope was developed towards the end of the sixteenth and the early years of the seventeenth century. The invention of the telescope (1609) and microscope (1610) has been accredited to Galileo, but it is possible that they had been built and used by others before then. Time and the discovery of the laws of optics have enabled numerous other optical instruments to be developed since that time. The early instruments were crude, and without an understanding of the laws of optics, it was difficult to design and build instruments that gave good

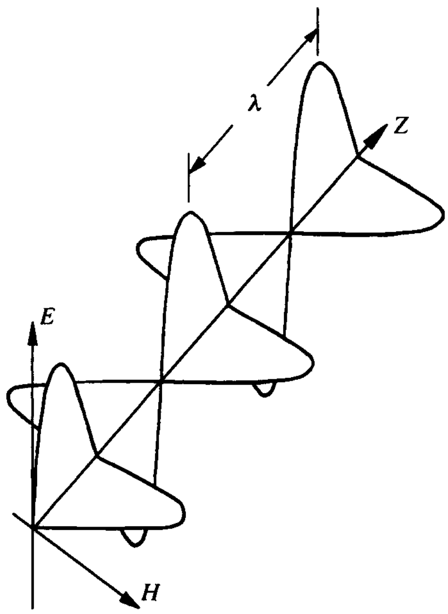


Fig. 1.1: The transverse electric and magnetic components of electromagnetic wave motion.

quality images. Now that we understand these laws, we can design and build optical instruments that give images of the highest possible quality.

While the use of visual optical instruments allows us to see far beyond the limitations of our eyes alone, these instruments also have limitations which are set by the laws of optics and the properties of light, in particular the wavelength of light. We will see that it is not possible to see objects smaller than the wavelength of the illuminating beam of radiation. As a rule, an object can only be "seen" by radiation whose wavelength is less than the object dimension. Therefore to "see" smaller and smaller detail, we must use shorter and shorter wavelengths, for example the X-ray microscope. However, since our eyes cannot respond to X-rays, we need to convert the X-ray image into a visible image.

The aim of this book is to describe the optical properties and functions of a wide range of visual optical instruments. To appreciate these aspects fully, we need to understand the nature of light and some of its basic properties. We will use the remainder of this chapter to cover this material, starting with the nature of light.

## 1.1 Electromagnetic radiation

Light is only a very small part of the electromagnetic radiation spectrum. Away from the source, electromagnetic radiation is a transverse wave motion composed of an electric ($E$) and a magnetic ($H$) field. These two fields are mutually perpendicular and also perpendicular to the direction of propagation, as shown in Figure 1.1. For this reason, electromagnetic radiation is sometimes called **transverse electromagnetic radiation** or wave motion.

Generally in optics, only the electric field component is important. This is because the interaction of electromagnetic radiation with matter involves interaction of the radiation with the electrons in the material, and whereas the electric field interacts with all electrons, the magnetic field only interacts with fast moving electrons. The electrons in optical materials are usually moving sufficiently slowly that their speed can be neglected and therefore they are only affected by the electric field component of the radiation.

Thus for our purposes, the transverse electromagnetic radiation can be sufficiently described in terms of the electric wave motion alone. For plane wave motion in a vacuum, at a distance $z$ from some arbitrary origin and at a time $t$, the electric field $E(z, t)$ can be described by the equation

$$E(z, t) = E_o \cos[2\pi(z/\lambda + \nu t + \delta)] \tag{1.1}$$

and in the complex algebra notation, it can be expressed in the form

$$E(z, t) = \text{real part of } E_o e^{[i2\pi(z/\lambda+\nu t+\delta)]}$$

where $E_0$ is the **amplitude** of the electric field, $\lambda$ is the **wavelength**, $\nu$ is the temporal frequency and $\delta$ is an arbitrary phase factor. However, we usually omit the reference to the real part and simply write

$$E(z, t) = E_o e^{[i2\pi(z/\lambda+\nu t+\delta)]} \tag{1.1a}$$

where the real part is assumed. The direction of the electric vector, or field or plane containing it, is called the direction of the plane of **polarization**.

Physical detectors of light, such as a light meter or the eye, cannot detect the instantaneous electric field. Instead they detect the time averaged square of the field. If we square the instantaneous electric field function $E(z, t)$ given by equation (1.1), carry out a temporal summation by integration with respect to $t$ and finally determine the average value for an infinite integration time, the final value is simply $E_o^2$. This quantity is often called the **intensity** as opposed to the amplitude. One advantage of the complex representation above is that the intensity is equivalent to the product of the complex field and its complex conjugate. That is,

$$\text{intensity} = E(z, t)E^*(z, t) = E_o^2 \tag{1.2}$$

where $E(z, t)$ is the complex electric field given by equation (1.1a) and $E^*(z, t)$ is the complex conjugate of $E(z, t)$, which is the same function as $E(z, t)$ except that the complex quantity $i[= \sqrt{(-1)}]$ is replaced by $-i$.

In a vacuum, the wavelength $\lambda$ and frequency $\nu$ are connected by the following equation:

$$\lambda\nu = c \tag{1.3}$$

where $c$ is the speed or velocity of propagation of the electromagnetic radiation in a vacuum. Its value is given in the summary of symbols at the end of the chapter.

In the visible part of the spectrum, the wavelength ranges from about 400 to 780 nm, with corresponding frequencies of $7.25 \times 10^{14}$ to $3.72 \times 10^{14}$ Hz. The wavelengths and frequencies of the different components of the electromagnetic spectrum are shown in Figure 1.2.

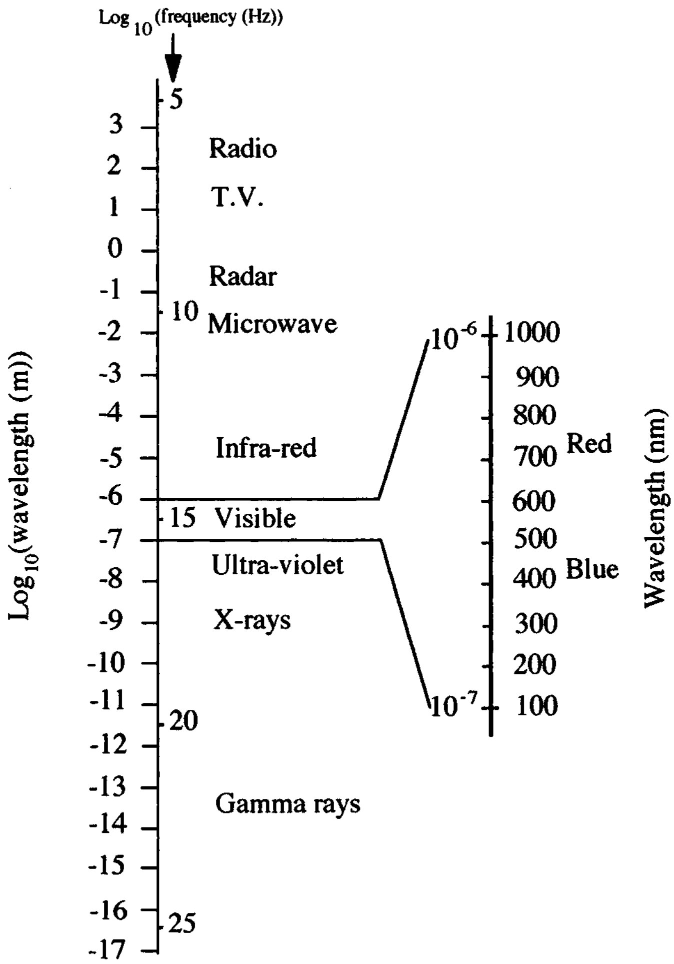


Fig. 1.2: The electromagnetic spectrum.

### *1.1.1 Particle or quantum theory*

While light has the properties of a wave motion, under some circumstances, it behaves like a stream of particles. This behaviour is embodied in the quantum theory. In the quantum theory description of electromagnetic radiation, the radiation is quantized into discrete energy packets called photons. The energy $E$ of a photon is given by the simple equation

$$E = h\nu = hc/\lambda \tag{1.4}$$

where $h$ is Planck's constant. Its value is given in the summary of symbols at the end of the chapter.

## 1.2 Refractive index and dispersion

The velocity of propagation of electromagnetic radiation through a medium depends upon how strongly it interacts with the charged particles in the medium. The refractive index ($n$) is a measure of the propagation velocity through the

*Table 1.1. The approximate refractive indices of some common materials*

| Material | Index |
|---|---|
| Air (15° and 76 cm Hg) | 1.00028 |
| Benzene | 1.50 |
| Dense flint | 1.625 |
| Diamond | 2.419 |
| Perspex | 1.490 |
| Sapphire | 1.77 |
| Sodium chloride | 1.54 |
| Water | 1.333 |
| White ophthalmic crown glass | 1.523 |
| Silicon | 4.000 (approx.) |

medium. It is defined as

$$n = \frac{\text{velocity in a vacuum } (c)}{\text{velocity in a material } (v)} = \frac{c}{v} \tag{1.5}$$

For any medium, the velocity $v$ is less than that in a vacuum. Therefore, the refractive index of any medium other than a vacuum is greater than 1.0. The above index is the **absolute refractive index**. However, most of the time we use the **relative refractive index**, that is the index relative to air and not to vacuum. Since the refractive index of dry air under normal conditions is close to unity ($n = 1.0003$), the absolute indices are about 0.03% higher than the corresponding relative indices. Because this difference is small, we often take the index of air as 1.0 and do not make a distinction between the relative and absolute index. Typical values of the refractive index of some common materials are given in Table 1.1.

### *1.2.1 Dependency of wavelength and frequency on refractive index*

Because there is a change of propagation speed or velocity when light enters a medium, there is a corresponding change in wavelength; therefore we can denote the wavelength dependency as $\lambda(n)$. For a medium with a refractive index $n$, the wavelength $\lambda(n)$ and frequency $\nu$ are connected by the equation

$$\lambda(n)\nu = v \tag{1.6}$$

which has the same form as equation (1.3). It follows from this equation that

$$\lambda(n)\nu = c/n = \lambda\nu/n$$

where $\lambda$ is the wavelength in vacuum. Therefore

$$\lambda(n) = \lambda/n \tag{1.7}$$

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More information



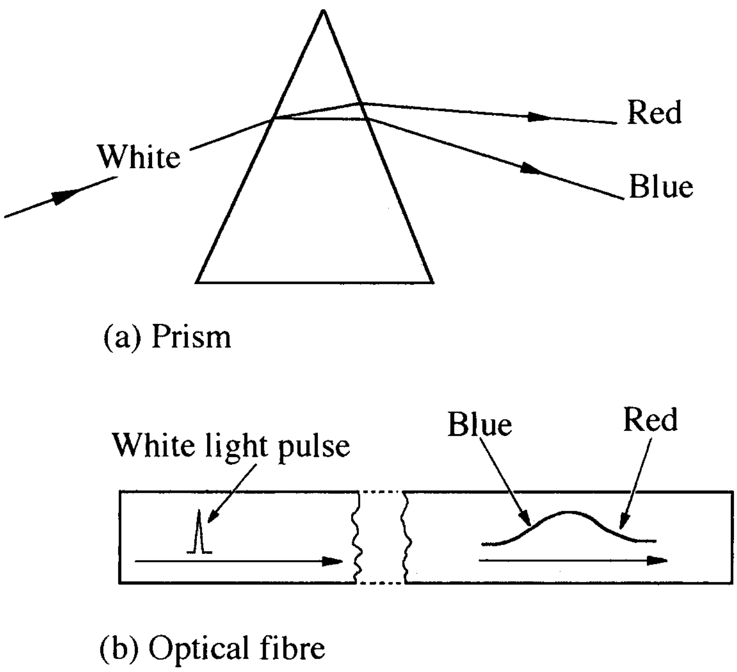


Fig. 1.3: Examples of the effect of dispersion.

However, while wavelength changes with refractive index, frequency does not change. Because of the constancy of frequency, it is common in many circumstances to specify a particular part of the electromagnetic radiation by its frequency rather than by its wavelength.

### *1.2.2 The dependence of refractive index on wavelength (dispersion)*

The refractive index varies with wavelength and, as a general rule, the refractive index decreases with increase in wavelength. The dependency of refractive index on wavelength is called **dispersion**.

The term dispersion is used because under many circumstances, the variation of refractive index with wavelength leads to a white light beam being broken up into its spectral colours. For example, if a beam of light passes through a prism, the beam is deviated through an angle which increases with increase in the refractive index of the prism. Now if a beam of white light passes through a prism as shown in Figure 1.3a, rays of different wavelengths are deviated by different amounts. Since the refractive index for blue light is higher than for red, the blue wavelengths are deviated through a greater angle. Thus the beam is dispersed into an angular distribution. The **rainbow** is due to a similar process in raindrops. A second example is the following. If a very short pulse of white light is sent into a long length of material such as an optical fibre as shown in Figure 1.3b, different wavelengths will travel at different velocities, thus lengthening or dispersing the pulse in the direction of travel. Because the index increases with a decrease in wavelength, red light is at the front of the pulse and the blue light is at the rear.

Because of dispersion, any stated refractive index should also be accompanied by the corresponding wavelength. For example it is common to specify the refractive index of optical glass at the wavelength $\lambda = 587.6$ nm, which is the yellow spectral line of helium. Sometimes average values, say over the visible spectrum, are used instead. The values given in Table 1.1 are mostly average values.

For glass and other common transparent optical materials, the variation of index can be accurately determined by a number of simple mathematical formulae. One due to Cauchy (1836) is of the form

$$n(\lambda) = A + B/\lambda^2 + C/\lambda^4 \tag{1.8a}$$

A second, known as the Hartmann equation (see Longhurst 1973, 500; Smith 1990, 164), is

$$n(\lambda) = n_o + A/(\lambda - \lambda_o)^{1.2} \tag{1.8b}$$

The values of the coefficients $A, B, C, n_o$ and $\lambda_o$ depend on the actual material and must be determined experimentally. A third equation is Sellmeir's dispersion formula (Born and Wolf, 1989, 97). This is commonly used by the manufacturers of optical glass, e.g. Schott (1992), and is as follows:

$$n^2(\lambda) - 1 = \frac{B_1\lambda^2}{\lambda^2 - C_1} + \frac{B_2\lambda^2}{\lambda^2 - C_2} + \frac{B_3\lambda^2}{\lambda^2 - C_3} \tag{1.8c}$$

where the values of $B_1$, $B_2$, $B_3$, $C_1$, $C_2$ and $C_3$ depend upon the particular glass type, which is given by Schott for each glass in their glass catalog. The accuracy of this equation is claimed to be $\pm 0.00001$ over the spectral range at which the refractive indices are specified. Another dispersion equation that has been used is

$$n^2(\lambda) = a_0 + a_1\lambda^2 + a_2/\lambda^2 + a_3/\lambda^4 + a_4/\lambda^6 + a_5/\lambda^8 \tag{1.8d}$$

#### 1.2.2.1 Quantification of dispersion

Like the refractive index, the dispersion of a material is a very important property of that material and expressing it as a single numerical value gives optical workers some immediate idea of how the refractive index of a material varies with wavelength. There are two common ways of quantifying dispersion. One is the Abbe $V$-value, often denoted by the symbol $V_d$. This is defined by the equation

$$V_d = \frac{(n_d - 1)}{(n_F - n_C)} \tag{1.9}$$

where

$n_d$ = is the refractive index at $\lambda = \lambda_d$

$n_F$ = is the refractive index at $\lambda = \lambda_F$

and

$$n_C = \text{is the refractive index at } \lambda = \lambda_C$$

where $\lambda_d$, $\lambda_F$ and $\lambda_C$ are wavelengths of certain spectral lines of gaseous elements, with $\lambda_d$ being in the middle of the visible spectrum (yellow) and the other two being towards the edges; $\lambda_F$ is blue and $\lambda_C$ is red. Their values are given in the symbols section at the front of the book. The $V$-value is sometimes defined for a different set of wavelengths, but we will use the above definition throughout this book and for simplicity mostly refer to it as the $V$-value and not as the $V_d$-value. A second way is in terms of the difference $(n_F - n_C)$ and this is called the **principal dispersion**.

The magnitude of the dispersion depends upon the type of material and as a general rule, the higher the refractive index the greater the dispersion. For most optical glasses, the values of $V_d$ are in the range of about 25 to 65, with the trend that the higher the refractive index the lower the $V$-value. Water has a $V$-value of about 55. Note that with the definition given by equation (1.9), the higher the dispersion, the lower the value of $V_d$. The corresponding values of the principal dispersion are in the range of about 0.03 to 0.008.

### *1.2.3 Gradient index materials*

The refractive index of most materials encountered in conventional optics is nominally constant throughout the bulk of the material. However, there are some materials in which the refractive index changes within the material, sometimes in a regular manner, and these are called gradient index materials. Two common naturally occurring examples are the atmosphere and the crystalline lens of the eye. Perhaps the most important man-made example is the gradient index optical fibre which is used in telecommunication transmissions. Gradient index fibres are briefly referred to again in Chapter 19.

## 1.3 Waves and rays

As mentioned in Section 1.1, there are two theories or descriptions of the nature of light. These are the wave theory and the particle theory. Both are valid but only when correctly applied. As a general rule, while light is travelling, the wave theory is used, but on interaction with materials, particularly when there is some absorption, the particle theory is usually applicable.

### *1.3.1 The wave theory*

If a point source of light is radiating light in all directions in an isotropic medium, the radiation field, for each wavelength in the source, can be pictured as a set of spherical **wavefronts** expanding outwards. The wavefronts are often regarded as the crests or troughs of the waves but any other phase of the wave may be used to define the wavefronts. Thus in a set of wavefronts, each neighbouring pair of wavefronts is one wavelength apart. In a more general situation, the wavefronts will be more complex in shape, for example if the propagation velocity or refractive index varies within the medium.

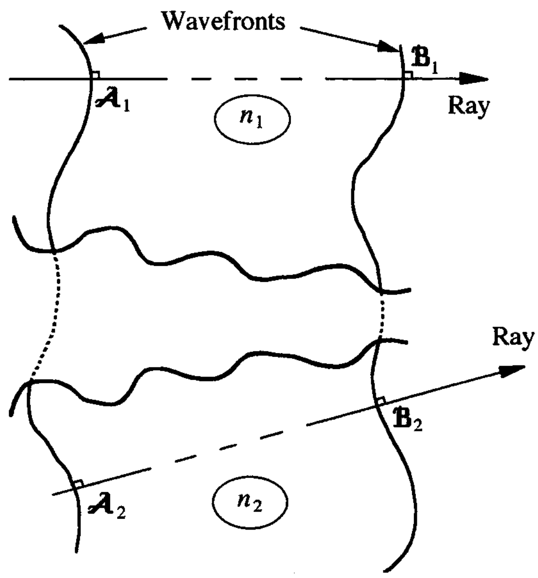


**Fig. 1.4:** Wavefronts and rays. The rays are the paths of normals to the wavefronts.

### *1.3.2 Rays*

Light rays are a useful concept to be used to trace the paths of the beams of radiation. Rays are imaginary lines drawn perpendicular to the wavefronts or may be interpreted as the expected paths travelled by particles (quanta or photons) of radiation. However, rays as paths of the wave normals is a much more useful interpretation. In a general situation, the wavefronts may not be spherical and instead may be of any shape, as shown in Figure 1.4. Two typical rays are shown in this diagram.

Let $\mathcal{A}_1\mathcal{A}_2$ and $\mathcal{B}_1\mathcal{B}_2$ be two successive wavefronts and $\mathcal{A}_1\mathcal{B}_1$ and $\mathcal{A}_2\mathcal{B}_2$ be two rays joining these wavefronts. We also let the two rays pass through regions with different refractive indices, denoted by $n_1$ and $n_2$ respectively. Since two neighbouring wavefronts in a set of wavefronts are one wavelength apart, it follows from equation (1.7) that the distances $\mathcal{A}_1\mathcal{B}_1$ and $\mathcal{A}_2\mathcal{B}_2$ are wavelengths in the respective media and thus

$$\mathcal{A}_1\mathcal{B}_1 = \lambda/n_1 \quad \text{and} \quad \mathcal{A}_2\mathcal{B}_2 = \lambda/n_2$$

where $\lambda$ is the vacuum wavelength. Therefore

$$n_1\mathcal{A}_1\mathcal{B}_1 = n_2\mathcal{A}_2\mathcal{B}_2 \tag{1.10a}$$

The quantity

refractive index × physical distance

is called the **optical path length** or **optical distance**. Thus the distances $n_1\mathcal{A}_1\mathcal{B}_1$ and $n_2\mathcal{A}_2\mathcal{B}_2$ are the optical path lengths of the physical distances $\mathcal{A}_1\mathcal{B}_1$ and $\mathcal{A}_2\mathcal{B}_2$. We often denote optical path lengths or distances by the squared brackets [ ].