

## *CONTENTS*

<i>Preface</i>	<i>page ix</i>
<b>1 Fundamental properties of vorticity</b>	<b>1</b>
1.1 Relation between velocity and vorticity	1
1.2 Vorticity and rotation	6
1.3 Circulation	8
1.4 Vortex lines and tubes	8
1.5 The laws of vortex motion	10
1.6 Kelvin's circulation theorem	14
1.7 Cauchy's equations	16
1.8 Irrotational flow	17
1.9 Bernoulli's equation	18
<b>2 Singular distributions of vorticity</b>	<b>20</b>
2.1 Vortex jumps	20
2.2 Vortex sheets	25
2.3 Line vortices	33
2.4 Image vorticity	39
<b>3 Vortex momentum</b>	<b>46</b>
3.1 Vortex force and bound vorticity	46
3.2 Hydrodynamic impulse	48
3.3 Impulsive generation from rest	51
3.4 Effect of compressibility	53
3.5 Angular impulse	55
3.6 Effect of viscosity	56
3.7 Impulse of isolated vortices	56
3.8 Impulse of a line vortex	60
3.9 Vortex centroid	61
3.10 Impulse in two dimensions	62

vi	<i>Contents</i>	
3.11	Kinetic energy of vortices	67
3.12	Helicity	69
3.13	Axisymmetric motion with swirl	71
<b>4</b>	<b>Motion with surfaces</b>	<b>74</b>
4.1	Virtual momentum of a moving body	74
4.2	Virtual momentum and impulse	76
4.3	Virtual angular momentum	78
4.4	Two-dimensional motion with circulation	79
<b>5</b>	<b>Some applications</b>	<b>81</b>
5.1	Virtual mass	81
5.2	Attraction of vortices or bodies to walls	84
5.3	Force on a body in a non-uniform stream	87
5.4	Force on a body in an accelerated irrotational stream	89
5.5	Rotating bodies	89
5.6	Torque on a body in a non-uniform stream	91
5.7	Self-propulsion of a deformable body	92
5.8	Buoyant vortex rings	93
<b>6</b>	<b>Creation of vorticity</b>	<b>95</b>
6.1	Formation of a vortex sheet	95
6.2	Leading edge suction on the sheet	97
6.3	Approximate development of the sheet into a vortex pair	99
6.4	Formation of a vortex ring	103
6.5	Creation of circulation about a body	105
6.6	Generation of vorticity by flow separation	107
6.7	Accelerated flow past a wing	109
<b>7</b>	<b>Dynamics of line vortices in two-dimensional flow</b>	<b>116</b>
7.1	Stability of a polygonal array	116
7.2	Vortices near walls	119
7.3	Kirchhoff–Routh path function	123
7.4	Conformal mapping and the Kirchhoff–Routh path function	126
7.5	Stability of infinite periodic arrays	130
7.6	The Karman vortex street	133
7.7	Statistical mechanics of assemblies of line vortices	138
<b>8</b>	<b>Vortex sheets in two dimensions</b>	<b>141</b>
8.1	The Birkhoff–Rott equation	141
8.2	Kelvin–Helmholtz instability	142
8.3	The ill-posedness of vortex sheets	145

<i>Contents</i>	vii
8.4 Roll-up of a semi-infinite vortex sheet: The Kaden spiral	147
8.5 General similarity solutions, single and multibranching spirals	152
<b>9 Dynamics of two-dimensional vortex patches</b>	<b>160</b>
9.1 Vortex sheets of finite thickness	160
9.2 Contour dynamics and Schwarz functions	163
9.3 The Kirchhoff vortex and elliptical patches in uniform strain	167
9.4 Equilibrium configurations for single patches	171
9.5 Filamentation	175
9.6 Vortex pairs	179
9.7 Arrays of vortex patches	183
<b>10 Axisymmetric vortex rings</b>	<b>192</b>
10.1 Formulation	192
10.2 Thin cored rings	195
10.3 Lamb's transformation and general core structure	201
10.4 Canonical co-ordinates for thin rings	206
<b>11 Dynamics of vortex filaments</b>	<b>209</b>
11.1 Local induction approximation	209
11.2 The cut-off method	212
11.3 Kelvin waves on a filament	215
11.4 Justification of the cut-off and higher-order approximations	218
<b>12 Three-dimensional vortex instability</b>	<b>230</b>
12.1 Oscillations of a vortex column	230
12.2 Long-wave co-operative instabilities	235
12.3 Short-wave co-operative instability	241
12.4 Ultra short-wave co-operative instability	250
<b>13 Effects of viscosity</b>	<b>253</b>
13.1 Viscous cores	253
13.2 Decay of trailing vortices	257
13.3 Burgers vortices	264
<b>14 Miscellaneous topics</b>	<b>271</b>
14.1 Minimum induced drag	271
14.2 Kelvin's variational principle	273
14.3 Hamiltonian dynamics of vortex patch moments	278

viii	<i>Contents</i>	
14.4	<b>Vortex breakdown</b>	285
	<i>Epilogue</i>	295
	<i>References</i>	296
	<i>Index</i>	309