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978-0-521-47739-0 - Vortex Dynamics
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The emphasis in this monograph is on the classical theory of inviscid incompressible fluids containing finite regions of vorticity. The effects of viscosity, compressibility, inhomogeneity, and stratification are enormously important in many fields of application, from hypersonic flight to global environmental fluid mechanics. However, this volume focuses on those aspects of fluid motion which are primarily controlled by the vorticity and are such that the effects of the other fluid properties are secondary.

Cambridge Monographs on Mechanics and Applied Mathematics

VORTEX DYNAMICS

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CAMBRIDGE
UNIVERSITY PRESS

Published by the Press Syndicate of the University of Cambridge
The Pitt Building, Trumpington Street, Cambridge CB2 1RP
40 West 20th Street, New York, NY 10011-4211, USA
10 Stamford Road, Oakleigh, Melbourne 3166, Australia

First published 1992
First paperback edition with corrections 1995

Library of Congress Cataloging-in-Publication Data

Saffman, P. G.
Vortex Dynamics / P.G. Saffman
p. cm. – (Cambridge monographs on mechanics and applied mathematics)
ISBN 0-521-42058-X (hc)
1. Vortex-motion. I. Title. II. Series.
QA925.S28 1992
532′.0595—dc20 92-9311
CIP

A catalog record for this book is available from the British Library.

ISBN 0-521-42058-X hardback
ISBN 0-521-477395 paperback

Transferred to digital printing 2001

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PREFACE

In the past three decades, the study of vortices and vortex motions – which originated in Helmholtz's great paper of 1858, 'Uber Integrale der hydrodynamischen Gleichungen welche den Wirbelbewegungen entsprechen' (translated by Tait [1867]), and continued in the brilliant work of Lord Kelvin and others in the nineteenth century, and Prandtl and his Göttingen school in the first half of this century – has received continuing impetus from problems arising in physics, engineering and mathematics. As aptly remarked by Küchemann [1965], vortices are the 'sinews and muscles of fluid motions'. Aerodynamic problems of stability, control, delta wing aerodynamics, high lift devices, the jumbo jet wake hazard phenomenon, among other concerns, have led to a myriad of studies. Smith [1986] reviews some of this work. The realisation that many problems involving interfacial motion can be cast in the form of vortex sheet dynamics has stimulated much interest. The discovery (rediscovery?) of coherent structures in turbulence has fostered the hope that the study of vortices will lead to models and an understanding of turbulent flow, thereby solving or at least making less mysterious one of the great unsolved problems of classical physics. Vortex dynamics is a natural paradigm for the field of chaotic motion and modern dynamical system theory. It is perhaps not well known that the father of modern dynamics and chaos wrote a monograph on vorticity (Poincaré [1893]). The theory of line vortices and vortex rings is a part of modern macroscopic treatments of liquid helium II, as is described by Donnelly and Roberts [1974]. Even Kelvin's [1867a] vortex theory of matter may one day have some topical interest.

Much of the modern work has been made possible by the remarkable advances in experimental techniques and the development of powerful computers, coupled with progress in numerical methods. Experiments and computations beyond the dreams of previous generations of students of

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vorticity are now routine, as we have tools with a power far greater than those available to the great scientists who founded the subject. Of course, this is not an unmixed blessing, as data and information are produced at a rate greater than the capability of the average scientist to absorb it. Indeed, the need for deep insight and analysis is more crucial than ever, in order to channel the energy available into productive research. As Kelvin [1880] remarked over 100 years ago, 'Crowds of exceedingly interesting cases present themselves'. We do not have the time, energy or resources for all members of this crowd, and the choice is overwhelming. Fortunately, analysis is still alive and helps to keep order, besides producing some of the important recent discoveries, such as the finite time vortex sheet singularity formation demonstrated by Moore [1979]. A powerful symbiosis between numerics and analysis has resulted in synergistic effects. Lamb's [1932 §159a] pessimistic remark, 'The motion of a solid in a liquid endowed with vorticity is a problem of considerable interest, but is unfortunately not very tractable', has lost much of its force. Vortex dynamics is a field of active research.

During the past twenty years, I have been teaching a course to graduate students in applied mathematics, aeronautics and engineering at Caltech on the fundamentals of vortex dynamics. An overview was published by Saffman and Baker [1979]. The present monograph is based on the notes of this course, and I am grateful to Prof Ari Glezer for providing me with a set of class notes more complete and in better order than those from which I lectured, and to Prof Hans Hornung who emphasised the pedagogical need for a text. In a field as well studied as vorticity, with a wealth of applications and a plethora of topics to investigate, it is difficult if not impossible to be both a scholar and an active researcher. If research is given priority over scholarship, then regretfully but inevitably the references to the corpus of work will be incomplete. This applies even more to papers and books not in the English language. In particular, there is outstanding research in the Soviet literature which is not properly acknowledged. A recent bibliography by Soviet workers (Akhmetov et al. [1988]) for the period 1975–87 contains 852 papers and is still not complete.

The emphasis in this monograph is on the classical theory of inviscid incompressible fluids containing finite regions of vorticity. Effects of viscosity, compressibility, inhomogeneity, stratification, and the like are enormously important in many fields of application, from hypersonic flight to global environmental fluid mechanics. But the discussion here focuses on those aspects of fluid motion which are primarily controlled by the vorticity and are such that the effects of the other fluid properties are secondary. It was my hope to present an up-to-date version of Chapter VII of Lamb's *Hydrodynamics* [1932], which is in my opinion (but not, I admit, in everybody's) unequalled in its combination of scholarship and research (although even Lamb omits some significant topics), but Lamb is not easily imitated or followed. There is no discussion here of the numerical methods for the computation of unsteady vortex flows such as the so-called vortex methods; this is more appropriately left to works on numerical analysis. Also omitted is any consideration of boundary layer theory, which can be and is treated as a strong interaction between the convection by external flow fields and its own self-induced velocity of the vorticity produced by viscous diffusion from a wall; boundary layer separation is then the highly non-trivial matter of determining where and under what conditions vorticity is convected away from the boundaries.

I should like to take this opportunity to thank the individuals and agencies who have supported my research on vorticity during the past twenty years and are therefore to some extent responsible for this monograph. The concern of Milt Rogers of the Air Force Office of Scientific Research over the jumbo jet, wake-hazard problem was instrumental in awakening much of the interest in vorticity in the 1960s and in starting a research programme at Caltech. Subsequently, support was provided by Ralph Cooper and Bob Whitehead of the Office of Naval Research and Milt Rose and Don Austin of what is now the Department of Energy. I owe a debt to all the scientific colleagues who participated in arguments and discussions and suggested many ideas; I have tried to acknowledge specific contributions. In particular, I wish to acknowledge an extensive, fruitful collaboration during the past twenty years with Prof Derek Moore.

Equations are numbered consecutively starting with 1 in each section. If the equation being referred to is in a different section of the same chapter, the section number is given first in arabic numerals separated by a dot from the equation number. If it is in a different chapter, the chapter number is given first in arabic numerals, followed, after a dot, by the section number. Sections are referred to by number if in the same chapter, and prefixed by the chapter number in arabic numerals if in a different chapter.