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LECTURES ON  
ARAKELOV GEOMETRY

C. SOULÉ

*CNRS,*

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written with

D. ABRAMOVICH, J.-F. BURNOL & J. KRAMER



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## Foreword

In the first semester of 1989, I was invited to give a graduate course in the Mathematics Department of Harvard University. Notes were written by D. Abramovich, J.-F. Burnol and J. Kramer, and typed in  $\text{T}_{\text{E}}\text{X}$  by L. Schlesinger in that department. They were revised and expanded during the next two years. My coauthors corrected and completed the contents of the book, especially by providing some of the proofs in Chapter VI. I also benefited from the advice of E. Getzler, who let me use a preliminary draft of his book with N. Berline and M. Vergne [BGV]. M. Hindry and A. J. Scholl read the manuscript carefully, and offered useful comments.

Without the efficient and generous help of all these people, these notes would not exist. I thank them very warmly, as well as the Harvard Mathematics Department for its hospitality.

C. Soulé.

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The material presented in this book is joint work with H. GILLET.  
Chapters VI and VII describe joint work with H. GILLET and J.-M.  
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