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0521475724 - Hyperbolicity and Sensitive Chaotic Dynamics at Homoclinic
Bifurcations: Fractal Dimensions and Infinitely Many Attractors

Jacob Palis and Floris Takens

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