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Ronald Meester and Rahul Roy
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This book is the first systematic and rigorous account of continuum percolation. The authors treat two models, the Boolean model and the random connection model, in detail, and discuss a number of related continuum models. Where appropriate, they make clear the connections between discrete percolation and continuum percolation.

All important techniques and methods are explained and applied to obtain results on the existence of phase transitions, equality of certain densities, continuity of critical densities with respect to distributions, uniqueness of the unbounded component, covered volume fractions, compression, rarefaction and so on. The book is self-contained, assuming only familiarity with measure theory and basic probability theory. The approach makes use of simple ergodic theory, but the underlying geometric ideas are always made clear.

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RONALD MEESTER
University of Utrecht

RAHUL ROY
Indian Statistical Institute

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Preface

This is the first book completely devoted to continuum percolation. The idea to write this book came up after we noticed that even specialists working in the larger area of spatial random processes were unaware about the current state of the art of continuum percolation. Although stochastic geometers have extensively studied the Boolean model, which is one of the most common models of continuum percolation, their focus has been on geometric and statistical aspects rather than on percolation-theoretical issues.

Initially, we planned to write a review article, but it became clear very quickly that it would be impossible to cover even the most basic results in such a review. Also it became apparent that it would be impossible to include in one volume all available results of a subject this size and still expanding. Therefore, we decided on a book which would give attention to all major issues and techniques without necessarily pushing them to the frontier of today's knowledge. When there is more to say on a specific subject than is found here, we provide the appropriate references for further reading.

Continuum percolation models are easily described verbally, but unlike discrete percolation models, their formal mathematical construction is not completely straightforward. In fact, many people (the authors included) have been quite careless with these constructions in the literature. The setup we have chosen in this book is probably the simplest rigorous construction which allows us to use all the ergodic theory we want. Perhaps some people will be bothered by the fact that we define ergodicity in terms of discrete group actions rather than as a continuum, but we prefer to avoid measure-theoretical nightmares in a book which is supposed to be on percolation theory and in which ergodicity is just a tool to work with. This rigorous construction is more for the mathematical completeness of the book. The reader may easily understand the book with just the geometric notion of the models in mind.

If we compare the results in this book to well-known results in discrete percolation models, we can roughly distinguish three classes of results. The first class of results consists of those for which there is a natural analogue in discrete percolation. For this class of results, the reader will notice that there are usually extra technical complications because we work in the continuum. These complications are often topological in nature or have something to do with the dependency structure in the models. As a result, statements are usually not quite the same as their discrete counterpart. Examples in this class are the non-triviality of phase transitions, the RSW lemma, the uniqueness of unbounded components and the equality of certain critical densities in random connection models. The second class of results consists of those whose discrete counterpart is either false or unknown. Examples in this class are the fact that the two most natural critical densities in certain Boolean models are not equal, the possible non-uniqueness of the unbounded component in certain cases and some limit results in continuum fractal percolation. And then, of course, there is a class of results which do not have a discrete analogue at all and which are special features of continuum models. This class, fortunately, is quite large and contains all high-density results like compression and rarefaction, results on the covered volume fraction, continuity of the critical densities when the radii distributions converge weakly in Boolean models, scaling properties and complete coverage results. In discrete models, the 'dual' structure plays an important role and can be described independently of the original structure. In continuum models, the vacancy structure plays a role similar to that of the dual, but the vacancy structure can only be described through the occupancy structure as its complement.

We have tried to make the book as self-contained as possible. It helps if the reader is familiar with discrete percolation theory, but this is not really necessary. The reader should feel comfortable with measure theory and basic probability theory, including branching processes. In order to avoid references scattered throughout the text, we conclude each chapter with notes which contain background information and references to related material.

Finally, we would like to thank Olle Häggström, Remco van der Hofstad, Karin Nelander, Mathew Penrose, Anish Sarkar and Jeffrey Steif for many comments on drafts of this book varying from correcting language to pointing out serious mathematical mistakes. Several visits of Rahul Roy to The Netherlands were partly financed by The Dutch Organisation of Scientific Research (N.W.O.). Ronald Meester would like to thank the people in the Indian Statistical Institute in New Delhi for their hospitality during several visits.

October 1995

Ronald Meester
Rahul Roy