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Symbol Index

Symbol	Name and Page	Symbol	Name and Page
A	set of accessible points, 206	$d_{\beta}^*(E, F)$	conjugate extremal distance, 131
A^∞, A^p	weights, 247	$d(z)$	distance from z to the boundary, 350
$A(\Omega, \rho)$	ρ -area, 130	$d_n(E)$	diameter of order n , 466
$AF(\zeta)$	area function of F , 348	$d_\infty(E)$	transfinite diameter, 467
$\alpha_\omega(z)$	lower pointwise dimension, 341	$\delta(z_0, E)$	reduced extremal distance, 162
$\beta(\Omega, \zeta_1, \zeta_2)$	β numbers, 290	diam	diameter
$\beta_E(z, t)$	362	$\dim(E)$	Hausdorff dimension, 457
$\beta_E(aQ)$	363	$\dim \omega$	dimension of harmonic measure, 341
$\beta^2(E)$	Jones square sum, 367	$\text{dist}(A, B)$	euclidean distance, 17
$\beta_\varphi(t)$	integral means spectrum, 286	$\text{dist}_\Omega(z_1, z_2)$	euclidean distance in Ω , 165
B	the set where φ' has nontangential limit zero, 207	ds	arc length, 129
$B(t)$	universal integral means spectrum, 286	$\frac{ dz }{1- z ^2}$	hyperbolic metric, 16
$B(t)$	Brownian path, 477	$D(u)$	Dirichlet norm, 447
$B_b(t)$	bounded universal integral means spectrum, 305	$D(u, v)$	Dirichlet inner product, 447
$B(\zeta, \delta), B_\delta(\zeta)$	disc, 14, 48	Δ	Laplacian
\mathcal{B}	Bloch space, 229	E^*	circular projection, 86
\mathcal{B}_0	little Bloch space, 258	$E(f \mathcal{N})$	conditional expectation, 470
$\ g\ _{\mathcal{B}}$	Bloch norm, 229	\mathcal{F}	fundamental domain, 345 normal, 415
BMO	bounded mean oscillation, 34	\mathcal{G}	Fuchsian group, 415
BMO(\mathcal{M})	bounded mean oscillation, 478	$\gamma(E)$	Robin's constant, 74–75
\mathbb{C}	complex plane	$\gamma_G(E)$	Green capacity, 111
$C^\alpha, C^{k+\alpha}$	Lipschitz class, 52, 55, 62	$\Gamma_\alpha(\zeta)$	cone, 6
\mathbb{C}^*	extended plane,	$\Gamma_\alpha^h(\zeta)$	truncated cone in \mathbb{D} , 203
c_U	central point, 350	$\Gamma_\beta^s(\zeta, \theta)$	truncated cone in \mathbb{C} , 173
Cap(E)	logarithmic capacity of a compact set, 74–75 of a Borel set, 85	$\Gamma_\alpha(\zeta, U)$	cone relative to U , 350
$Cl(f, \zeta)$	cluster set, 469	$g_\Omega(z, w)$	Green's function with pole at w on a finitely connected domain, 41 on an arbitrary domain, 75
$\partial\Omega$	boundary of Ω ,	G	the set where a conformal map has non-zero angular derivative, 207
$\delta_{\mathbb{D}}(z_1, z_2)$	pseudohyperbolic metric, 23		
\mathbb{D}	unit disc, 5		
$d_\Omega(E, F)$	extremal distance, 130		

Symbol	Name and Page	Symbol	Name and Page
$G_\mu(z)$	Green potential on \mathbb{H} , 109 on \mathbb{D} , 111	Ω_d	doubled Riemann surface, 445
$h(r)$	measure function, 456	(Ω, E, F)	triple, 484
\mathbb{H}	upper half-plane, 1	\mathcal{P}_f	Perron solution to the Dirichlet problem, 73
H^∞	bounded analytic functions, 435	$P_z(t)$	Poisson kernel for \mathbb{H} , 4
H^p	Hardy space, 435	$P_z(\theta)$	Poisson kernel for \mathbb{D} , 5
$H^p(U)$	Hardy space on domain U , 349	$P_z(\zeta)$	Poisson kernel for $z \in \Omega$ at $\zeta \in \partial\Omega$, 45
$I(v)$	energy integral, 79	$P_z^*(\theta)$	smallest even decreasing majorant of $P_z(\theta)$, 9
Im	imaginary part	$P_j(\tilde{u})$	period of \tilde{u} , 443
$K = K(\Omega)$	set of cone points, 208	$\Phi(\alpha)$	universal dimension spectra, 341
K^*	growth rate of number of bad discs, 288	$PV f$	principal value integral, 488
$\ell(Q)$	length of the base of Q , 31	Q	box, 31
$\lambda_\Omega(\Gamma)$	extremal length, 130	$Q_\Omega(w_1, w_2)$	quasihyperbolic distance, 20
$\lambda_\Omega(\Gamma)^{-1}$	module, 133	\mathbb{R}	real numbers, 1
$\lambda(z_0, E)$	146	Re	real part
$\Lambda_2(E)$	two-dimensional Hausdorff measure, 456	$\rho(z_1, z_2)$	hyperbolic distance in \mathbb{D} , 16
$\Lambda_h(E)$	h -Hausdorff measure, 457	$\rho_\Omega(w_1, w_2)$	hyperbolic distance in Ω , 20
$\Lambda_\alpha(E)$	α -dimensional Hausdorff measure, 457	ρ	metric, 130
$ \cdot $	length or absolute value	ρ -length	ρ -length, 130
$\ f\ _{C^\alpha}$	Lipschitz norm, 52	$S(z, w)$	spherical distance, 258
$Mf(\zeta)$	Hardy–Littlewood maximal function, 9	$S_n(f)$	square function, 506
\mathcal{M}	σ -algebra, 470	$S\varphi(z)$	Schwarzian derivative, 380
\mathcal{M}	group of self maps of \mathbb{D} , 16	■	end of the proof
$m(\lambda)$	distribution function, 438	\mathbb{S}	standard strip, 176
$\text{mod}(\Omega)$	module of a ring domain, 151	$\text{supp } \mu$	closed support of μ , 96
$M(G)$	reduced modulus, 168	$T(Q)$	top half of Q , 107
$m_h(E)$	dyadic Hausdorff content, 458	$\theta(x)$	148
$M_h(E)$	Hausdorff content, 456	$\text{Tn}(\Gamma)$	tangent points of Γ , 370
μ_E	equilibrium distribution, on $C^{1+\alpha}$ Jordan curves, 78 on a compact set, 83	\tilde{u}	conjugate function or harmonic conjugate of u , 50
∇	gradient	$u_\alpha^*(\zeta)$	nontangential maximal function, 7
∇_2	2nd order gradient, 56	u_f	solution to the Dirichlet problem on \mathbb{H} , 3 (Poisson integral, 4) on \mathbb{D} , 7 (Poisson integral, 5) on a Jordan domain, 13 on a finitely connected Jordan domain, 38
$\eta_\Omega(z_\theta, t)$	385	$U_\mu(z)$	logarithmic potential, 77
$\ \cdot\ $	norm	VMO	vanishing mean oscillation, 268
n	unit outer normal, 43	VMOA	analytic and vanishing mean oscillation, 268
$N(\Omega, \varepsilon, \rho)$	number of bad discs, 281	(X, \mathcal{M}, P)	probability space, 470
$\omega(z, E, \mathbb{H})$	harmonic measure in half-plane, 2–4	(W, \mathcal{M}, P)	Brownian motion, 477
$\omega(z, E, \mathbb{D})$	harmonic measure in disc, 5	Z^*	Zygmund class, 56
$\omega(z, E, \Omega)$	harmonic measure, in a Jordan domain, 13 in a finitely connected Jordan domain, 39 in an arbitrary domain, 90	$\ f\ _{Z^*}$	Zygmund norm, 56
$\omega_f(\delta)$	modulus of continuity, 70	Z_n	exponential transform, 508
		\emptyset	empty set

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Definitions are found on **boldface** page numbers

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