

Index

- adjoint operator, *see* formal adjoint operator
- Assumption A 47
 Assumption E 79
 Assumption F 372
 Assumption H 85
 Assumption \tilde{H} 102
 Assumption H_{loc} 203
 Assumption \tilde{H}_{loc} 123
 Assumption SP 384
- barrier function 58
 Blumenthal 0–1 law 55
 boundary
 $C^{k,\alpha}$ 85
 regular points 55
 boundary Harnack principle 333
 bounded harmonic function 435
 for Laplacian on manifold 448, 452
 probabilistic characterization 436, 438
 Brownian motion 6
 on manifold 445
- Cameron–Martin–Girsonov formula 27
 canonical construction 43
 cemetery state 44
 Chapman–Kolmogorov equation 4
 compact operator 77
 comparison principle
 manifold of negative curvature 453
 Martin boundary 400–3
 transience and recurrence 256
 conditional expectation 2
 conditioned diffusion and h -transforms 286–291
 cone in a Banach space 78
 core 90
 coupling 458
 critical operator 145
 bounded domains 146, 176
 ground state 149
 invariance under h -transform 148
 one-dimensional case 211–12
 recurrence 148
 small perturbations 166, 168, 219, 220
 see also subcritical operator
 curvature 446
- diffusion 43
 Dirichlet principle 255
 Dirichlet problem 86
 domain
 $C^{k,\alpha}$ -boundary 85
 of infinitesimal generator 89
 Doob’s optional sampling theorem 3
- eigenvalue, *see* principal eigenvalue and generalized principal eigenvalue
 entrance time, *see* first entrance time
 escape rate of diffusion 160
 exit probability function 437
 exit time, *see* first exit time
 explosion 44
 criteria 212, 277, 278
 invariant densities and functions 182
 explosion inward from the boundary 376
 criteria 377
 smooth bounded domain 379
 exterior cone condition 55
 exterior harmonic measure boundary 320
 Martin boundary 322
 non-dependence on domain 325
 exterior sphere condition 55
- Feller process 5
 Feller’s test for explosion 212
 Feynman–Kac formula 60
 filtration 1
 first entrance time 67
 of a single point 144
 first exit time 30
 finite expectation 48, 49
 from a small ball 50
 formal adjoint operator 102
 generalized principal eigenvalue 148
 principal eigenvalue 102
 Fredholm alternative 78, 87, 247
 Fuchsian operator 371, 372
 adjoint 373
 Martin boundary 372

- gauge 121
- generalized Dirichlet principle 261, 262
- generalized martingale problem
 - for L on R^d 37
 - for L on D 42
- generalized maximum principle 81, 100, 179
- generalized principal eigenvalue 147
 - continuity properties 156, 157, 179
 - convexity 159, 179
 - criteria for finiteness or infiniteness 163
 - escape rate of diffusion 160
 - invariance under h -transform 148
 - mini-max characterization 160, 161
 - small perturbations 168, 219, 220
 - symmetric case 194, 195
- generator, *see* infinitesimal generator
- geodesic 445
- Green's function 136, 137
 - basic properties 136, 143
 - boundary behavior 333, 334
 - examples and estimates 142, 143, 305–9
 - Laplacian 142, 143
 - positive solutions of minimal growth 297
 - stochastic representation 294
- Green's measure 129, 130
 - transience and recurrence 130
- ground state 149
 - boundary behavior 336
 - examples and estimation 304–9
 - positive solutions of minimal growth 300
 - representation via Green's functions 300
- harmonic measure
 - h -transform 314–17
 - Lebesgue surface measure 311, 312, 317
 - mutual absolute continuity 310, 311
 - positive solutions of minimal growth 310
 - weak continuity 310
- Harnack's inequality 124
 - boundary, *see* boundary Harnack principle
- h -transform 126
 - conditional diffusions 286–91, 330, 331
 - Green's function 133
 - Martin boundary, *see* Martin boundary
 - transition measure 126
- Hopf maximum principle 84
- infinitesimal generator 89
- invariant density, function, measure 179
 - critical case 182, 183
 - example 216–18
 - explosion 182
 - one-dimensional case 215
 - supercritical case 180
- invariant set 435
- invariant σ -algebra 435
 - Brownian motion on a manifold 448, 452
 - triviality 436
- Itô's formula 15, 16
- Itô process 10
- Krein–Rutman theorem 79
- Laplace–Beltrami operator 155, 445
- Liapunov function method 236, 237
 - explosion 277–9
 - transience and recurrence 238–54
- lifetime of diffusion 43
- Liouville's theorem 457–8
- localization
 - martingale problem 29–31, 42, 43
 - stochastic differential equation 24
- localization of binding 171
- manifold, *see* Riemannian manifold
- Markov process 4
 - time homogeneous 4
- Martin boundary 283
 - Brownian motion on model manifold 448
 - conditioned diffusions 323, 420
 - convergence of h -transformed diffusion 286–91
 - exterior harmonic measure boundary 322
 - Fuchsian operators and their adjoints 372, 373
 - Laplacian in cone 391, 421
 - Laplacian in strip 392, 393, 421
 - Lipschitz domain 337, 338, 345, 346
 - minimal 285
 - period operator 362
 - Picard principle, *see* Picard principle
 - skew product form operator 384–405
- Martin kernel 283
- Martin representation theorem 285
- Martin sequence 283
- Martin topology 284
- martingale (sub, super) 2, 3
- martingale convergence theorem 3, 4
- martingale inequalities 3
- martingale problem
 - for L on R^d 26
 - for L on D 39
 - see also* generalized martingale problem

- maximum principle 80; *see also*
 - generalized maximum principle *and* strong maximum principle
- minimal growth, *see* positive solution of minimal growth
- minimal Martin boundary 285
- minimal positive harmonic function 284
 - Lipschitz domain 345, 346
- mini-max formula
 - Dirichlet problem 261, 262
 - generalized principal eigenvalue 160, 161
 - principal eigenvalue 102, 103
 - transience and recurrence 270, 271
- model manifold 446

- null recurrence, *see* positive recurrence and null recurrence

- optional sampling theorem, *see* Doob's optional sampling theorem

- periodic operator 347
 - criticality 352
 - Martin boundary 362
 - perturbation theory 363–5
 - positive harmonic functions 348–51, 354, 355
- perturbation theory 164–76, 219–31
- Picard principle 395, 396, 422, 423
- positive harmonic function 144
 - critical case 148, 149
 - invariant density and invariant function 181
 - Martin boundary, *see* Martin boundary
- positive operator 79
- positive recurrence and null recurrence 185
 - criteria 219, 236, 237, 240, 242–3, 245–7
 - ergodic properties 185, 186, 192
 - first entrance time to compact set 186
 - one-dimensional case 219
- positive solution of minimal growth
 - applications 304–9
 - characterization 296, 297
 - Green's function 297
 - ground state 300
 - stochastic representation 296, 297
 - subcritical case 302
- principal eigenvalue 95
 - asymptotic behavior of exit time 95
 - mini-max formula 102, 103
 - on small balls 115
- product L^1 criticality 185
- progressive measurability 1

- recurrence, *see* transience and recurrence
- regular boundary point 55
 - exterior cone condition 55, 56, 58, 75
- resolvent equation 152, 153
- resolvent set 77
- Riemannian manifold
 - criticality theory 196–200
 - model manifold 446
 - nonnegative curvature and Brownian motion 444–6
 - with ends 324
- Riesz–Schauder theorem 78

- Schauder global estimate 86
- Schauder interior estimate 86
- sectional curvature 444
- semigroup 5, 89
- shift operator 54
- σ -algebra
 - completion 1
 - up to τ 3
- skew product form 380
 - criticality 382
 - decomposition 380, 381
- small time behavior of diffusion 52
- spectrum 77
- standard compactness argument 147
- stochastic averaging 242, 244
- stochastic differential equation 17
- stochastic integral 15
- stochastic process 1
- stochastic representation of solutions 49, 60, 61, 100, 101
- stopping time 3
- strong Markov process 5
- strong maximum principle 84
- Stroock–Varadhan support theorem 65
- subcritical operator 145
 - inhomogeneous Dirichlet problem 151
 - invariance under h -transform 148
 - small perturbations 165
 - transience 148
 - see also* critical operator
- submartingale, *see* martingale
- supercritical operator 145
- supermartingale, *see* martingale
- support theorem, *see* Stroock–Varadhan support theorem
- symmetric operator 193, 194, 207, 208

- time change 46
- time homogeneous Markov process 4
- transience and recurrence
 - basic properties 68–75
 - Brownian motion on a manifold 447, 452

474

Index

- transience and recurrence (*cont.*)
 criteria 208, 209, 236, 237, 239, 242,
 243, 245–7, 256–8, 260–1, 271, 447,
 452
 first entrance time to compact set 73–5
 mini–max formula 271
 on $D \subseteq R$ 74
 on R^d 68
- one-dimensional case 208, 209
 variational approach 255–61
- transition measure 124
 behavior for large time 185, 186, 192
transition probability function 4
- weak convergence 309
Wiener measure 7