METHODS
OF
ALGEBRAIC GEOMETRY

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by

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VOLUME II

BOOK III: GENERAL THEORY OF ALGEBRAIC VARIETIES IN PROJECTIVE SPACE

BOOK IV: QUADRICS AND GRASSMANN VARIETIES

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PREFACE

THIS VOLUME gives an account of the principal methods used in developing a theory of algebraic varieties in space of n dimensions. Applications of these methods are also given to some of the more important varieties which occur in projective geometry. It was originally our intention to include an account of the arithmetic theory of varieties, and of the foundations of birational geometry, but it has turned out to be more convenient to reserve these topics for a third volume. The theory of algebraic varieties developed in this volume is therefore mainly a theory of varieties in projective space.

In writing this volume we have been faced with two problems: the difficult question of what must go in and what should be left out, and the problem of the degree of generality to be aimed at. As our objective has been to give an account of the modern algebraic methods available to geometers, we have not sought generality for its own sake. There is still enough to be done in the realm of classical geometry to give these methods all the scope that could be desired, and had it been possible to confine ourselves to the classical case of geometry over the field of complex numbers, we should have been content to do so. But in order to put the classical methods on a sound basis, using algebraic methods, it is necessary to consider geometry over more general fields than the field of complex numbers. However, if the ultimate object is to provide a sound algebraic basis for classical geometry, it is only necessary to consider fields without characteristic. Since geometry over any field without characteristic conforms to the general pattern of geometry over the field of complex numbers, we have developed the theory of algebraic varieties over any field without characteristic. Thus fields with finite characteristic are not used in this book.

As for the material included, the space factor has caused us to restrict ourselves to fundamental concepts and methods. But it is our hope that a reader who has mastered the methods described in these volumes, and has seen how they can be applied to some of the standard problems in geometry, will be able to apply them to more advanced geometrical problems.

Chapter X (the first in this volume) is devoted to definitions, and to the study of the basic concepts of the theory of algebraic varieties, including irreducibility, generic points, dimension and order. The principal tool employed in this chapter is the 'Zugeordnete Form', introduced by van der Waerden and Chow, to which we have given the name 'Cayley form', for historical reasons discussed in the text. The ideas of this chapter owe much to van der Waerden, but the greater prominence given here to the Cayley form has resulted in a somewhat different, and, we believe, more complete account of the basic concepts than he has yet given.

In Chapter XI we deal with the foundations of the theory of algebraic correspondences, and apply this theory to develop the idea of multiplicities in geometry. This chapter also owes much to van der Waerden, but, following Weil, we make a sharp distinction between point-set properties of varieties and multiplicative properties. Chapter XII begins with an account of the intersections of algebraic varieties, and then goes on to develop the algebraic theory of systems of varieties. The first three chapters of this volume are therefore mainly concerned with general theory, but the principles are illustrated by applications to examples. A particularly convenient example is afforded by the Segre variety which represents an r-way space as a variety in one-way space.

Chapters XIII and XIV deal with Quadrics and Grassmann Varieties respectively. The purpose of these chapters is to show how the general methods developed in earlier chapters can be used to develop the properties of these loci on a strictly algebraic basis. The account given in each case is not intended to be complete, although a considerable range of properties is reviewed. The methods used for finding a base on a number of the varieties may appear to be rather special, but no general method is known at present.

We again have to express our thanks to Prof. T. A. A. Broadbent of the Royal Naval College, Greenwich, for valuable assistance in the preparation of the manuscript and in the correction of proofs, and to the Cambridge University Press for their assistance in technical matters.

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Note. The reference [II, §4, Th. II] is to Theorem II in §4 of Chapter II of Vol. I, which contains Chapters I–IX. This volume contains Chapters X–XIV. If a reference is to the same chapter or section, the corresponding numeral or numerals are omitted.

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D. P.

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