METHODS
OF
ALGEBRAIC GEOMETRY

METHODS
OF
ALGEBRAIC GEOMETRY

by

W. V. D. HODGE, Sc.D., F.R.S.
Formerly Lowndean Professor of Astronomy and Geometry, and Master of Pembroke College, Cambridge

and

D. PEDOE, PH.D.
Emeritus Professor of Mathematics University of Minnesota

VOLUME I

BOOK I: ALGEBRAIC PRELIMINARIES
BOOK II: PROJECTIVE SPACE

CAMBRIDGE
UNIVERSITY PRESS

Published by the Press Syndicate of the University of Cambridge
The Pitt Building, Trumpington Street, Cambridge CB2 1RP
40 West 20th Street, New York, NY 10011–4211, USA
10 Stamford Road, Oakleigh, Melbourne 3166, Australia

First published 1947
Reprinted 1953, 1968
First paperback edition 1968
Reissued in the Cambridge Mathematical Library 1994

ISBN 0 521 469007 Paperback

Transferred to digital printing 1999

CONTENTS

BOOK II

PROJECTIVE SPACE

CHAPTER V: PROJECTIVE SPACE: ALGEBRAIC DEFINITION

CHAPTER VI: PROJECTIVE SPACE: SYNTHETIC DEFINITION

CHAPTER VII: GRASSMANN COORDINATES

CHAPTER VIII: COLLINEATIONS

CHAPTER IX: CORRELATIONS

PREFACE

THIS VOLUME is the first part of a work designed to provide a convenient account of the foundations and methods of modern algebraic geometry. Since nearly every topic of algebraic geometry has some claim for inclusion it has been necessary, in order to keep the size of this volume within reasonable limits, to confine ourselves strictly to general methods, and to stop short of any detailed development of geometrical properties.

We have thought it desirable to begin with a section devoted to pure algebra, since the necessary algebraic topics are not easily accessible in English texts. After a preliminary chapter on the basic notions of algebra, we develop the theory of matrices. Some novelty has been given to this work by the fact that the ground field is not assumed to be commutative. The more general results obtained are used in Chapters V and VI to analyse the concepts on which projective geometry is based. Chapters III and IV, which will be required in a later volume, are devoted to a study of algebraic equations.

Book II is concerned with the definition and basic properties of projective space of n dimensions. Both the algebraic and the synthetic definitions are discussed, and the theory of matrices over a non-commutative field is used to show that a space based on the propositions of incidence can be represented by coordinates, without the introduction of any assumption equivalent to Pappus' theorem. The necessity of considering a large number of special cases has made Chapter VI rather long, but some space has been saved in the later parts of the chapter by merely mentioning the special cases and leaving the proofs to the reader, when they are sufficiently simple. It is hoped that this will not cause any difficulty. This Book concludes with a purely algebraic account of collineations and correlations. Certain elementary geometrical consequences are indicated, but a complete study of the geometrical problems involved would have taken us beyond our present objective.

It is hoped that Volume II will appear shortly. This will be devoted to the theory of algebraic varieties, and to the study of certain loci which arise in many geometrical problems.

We wish to express our thanks to Professor T. A. A. Broadbent of the Royal Naval College, Greenwich, who has read this volume in manuscript and in proof, and to Mr D. B. Scott of the University of Aberdeen, who has read it in proof. We must also thank the staff of the University Press for the care they have taken in the production of this book, and for their ready courtesy in meeting our wishes.

Note. The reference [II, §4, Th. II] is to Theorem II in §4 of Chapter II. If the reference is to the same chapter or section, the corresponding numeral or numerals will be omitted.

W. V. D. H.
D. P.

CAMBRIDGE
November 1946