This book introduces the tools of modern differential geometry – exterior calculus, manifolds, vector bundles, connections – to advanced undergraduates and beginning graduate students in mathematics, physics, and engineering. It covers both classical surface theory and the modern theory of connections and curvature, and includes a chapter on applications to theoretical physics. The only prerequisites are multivariate calculus and linear algebra; no knowledge of topology is assumed.

The powerful and concise calculus of differential forms is used throughout. Through the use of numerous concrete examples, the author develops computational skills in the familiar Euclidean context before exposing the reader to the more abstract setting of manifolds. There are nearly 200 exercises, making the book ideal for both classroom use and self-study.

Differential Forms and Connections

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Preface

Purpose

This book represents an extended version of my lecture notes for a one-semester course on differential geometry, aimed at students without knowledge of topology. Indeed the only prerequisites are a solid grasp of multivariate calculus and of linear algebra. The goal is to train advanced undergraduates and beginning graduate students in exterior calculus (including integration), covariant differentiation (including curvature calculations), and the identification and uses of submanifolds and vector bundles. It is hoped that this will serve both the minority who proceed to study advanced texts in differential geometry, and the majority who specialize in other subjects, including physics and engineering.

Summary of the Contents

Every generation since Newton has seen a richer and deeper presentation of the differential and integral calculus. The nineteenth century gave us vector calculus and tensor analysis, and the twentieth century has produced, among other things, the exterior calculus and the theory of connections on vector bundles. As the title implies, this book is based on the premise that differential forms provide a concise and efficient approach to many constructions in geometry and in calculus on manifolds.

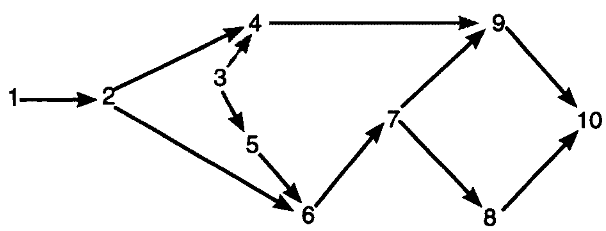
Chapter 1 is algebraic; Chapters 2, 4, 8, and 9 are mostly about differential forms; Chapters 4, 9, and 10 are about connections; and Chapters 3, 5, 6, and 7 are about underlying structures such as manifolds and vector bundles. The reader is not mistaken if he detects a strong influence of Harley Flanders's delightful 1989 text. I would also like to acknowledge that I have made heavy use of ideas from Berger and Gostiaux [1988], and (in Chapters 6 and 9) of my handwritten Warwick University 1981 lecture notes from John Rawnsley, as well as other standard differential geometry texts. Chapter 9 on connections is in the spirit of S. S. Chern [1989], p. ii, who remarks that "the notion of a connection in a vector bundle will soon find its way into a class on advanced calculus, as it is a fundamental notion and its applications are widespread"; these applications include the field theories of physics (see Chapter 10), the study of information loss in parametric statistics, and computer algorithms for recognizing surface deformation. Regrettably the Frobenius Theorem and its applications, and de Rham cohomology, are among many other topics which could not be included; see Flanders [1989] for an excellent treatment of the former, and Berger and Gostiaux [1988] for the latter.

Prerequisites

- **Linear Algebra:** finite-dimensional vector spaces and linear transformations, including the notions of image, kernel, rank, inner product, and determinant.
- **Vector Calculus:** derivative as a linear mapping; grad, div, and curl; line, surface, and volume integrals, including Green's Theorem and Stokes's Theorem; implicit function theorem; and the concept of an open set in Euclidean space.

Advice to the Instructor

In the diagram below, a solid arrow denotes dependency of chapters, and a fuzzy arrow denotes a conceptual relationship. In one semester, an instructor would probably be hard pressed to cover more than six chapters in depth. Chapters 1 and 2 are essential. Some instructors may choose to emphasize the easier and more concrete material in Chapters 3 and 4, which is used in the sequel only as a source of examples, while others may prefer to move rapidly into Chapters 5 and 6 so as to have time for Chapter 8 on integration and/or Chapter 9 on connections. Alternatively one could deemphasize abstract differential manifolds (i.e., skip over Chapter 5), cover only the "local vector bundle" part of Chapter 6, and treat Chapters 7 to 10 in a similarly "local" fashion. As always, students cannot expect to master the material without doing the exercises.



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