

Cambridge University Press
978-0-521-46713-1 - Topics in Matrix Analysis
Roger A. Horn and Charles R. Johnson
Frontmatter
[More information](#)

TOPICS IN MATRIX ANALYSIS

Cambridge University Press
978-0-521-46713-1 - Topics in Matrix Analysis
Roger A. Horn and Charles R. Johnson
Frontmatter
[More information](#)

Topics in matrix analysis

ROGER A. HORN
University of Utah

CHARLES R. JOHNSON
College of William and Mary



CAMBRIDGE
UNIVERSITY PRESS

Cambridge University Press
978-0-521-46713-1 - Topics in Matrix Analysis
Roger A. Horn and Charles R. Johnson
Frontmatter
[More information](#)

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore,
São Paulo, Delhi, Dubai, Tokyo

Cambridge University Press
32 Avenue of the Americas, New York, NY 10013-2473, USA

www.cambridge.org
Information on this title: www.cambridge.org/9780521467131

© Cambridge University Press 1991

This publication is in copyright. Subject to statutory exception
and to the provisions of relevant collective licensing agreements,
no reproduction of any part may take place without the written
permission of Cambridge University Press.

First published 1991
First paperback edition (with corrections) 1994
10th printing 2008

A catalog record for this publication is available from the British Library

ISBN 978-0-521-30587-7 Hardback
ISBN 978-0-521-46713-1 Paperback

Transferred to digital printing 2010

Cambridge University Press has no responsibility for the persistence or
accuracy of URLs for external or third-party internet websites referred to in
this publication, and does not guarantee that any content on such websites is,
or will remain, accurate or appropriate. Information regarding prices, travel
timetables and other factual information given in this work are correct at
the time of first printing but Cambridge University Press does not guarantee
the accuracy of such information thereafter.

Contents

Preface	<i>page vii</i>
Chapter 1 The field of values	1
1.0 Introduction	1
1.1 Definitions	5
1.2 Basic properties of the field of values	8
1.3 Convexity	17
1.4 Axiomatization	28
1.5 Location of the field of values	30
1.6 Geometry	48
1.7 Products of matrices	65
1.8 Generalizations of the field of values	77
Chapter 2 Stable matrices and inertia	89
2.0 Motivation	89
2.1 Definitions and elementary observations	91
2.2 Lyapunov's theorem	95
2.3 The Routh-Hurwitz conditions	101
2.4 Generalizations of Lyapunov's theorem	102
2.5 M -matrices, P -matrices, and related topics	112
Chapter 3 Singular value inequalities	134
3.0 Introduction and historical remarks	134
3.1 The singular value decomposition	144
3.2 Weak majorization and doubly substochastic matrices	163
3.3 Basic inequalities for singular values and eigenvalues	170
3.4 Sums of singular values: the Ky Fan k -norms	195
3.5 Singular values and unitarily invariant norms	203

vi	Contents	
	3.6	Sufficiency of Weyl's product inequalities 217
	3.7	Inclusion intervals for singular values 223
	3.8	Singular value weak majorization for bilinear products 231
Chapter 4		Matrix equations and the Kronecker product 239
	4.0	Motivation 239
	4.1	Matrix equations 241
	4.2	The Kronecker product 242
	4.3	Linear matrix equations and Kronecker products 254
	4.4	Kronecker sums and the equation $AX + XB = C$ 268
	4.5	Additive and multiplicative commutators and linear preservers 288
Chapter 5		The Hadamard product 298
	5.0	Introduction 298
	5.1	Some basic observations 304
	5.2	The Schur product theorem 308
	5.3	Generalizations of the Schur product theorem 312
	5.4	The matrices $A \circ (A^{-1})^T$ and $A \circ A^{-1}$ 322
	5.5	Inequalities for Hadamard products of general matrices: an overview 332
	5.6	Singular values of a Hadamard product: a fundamental inequality 349
	5.7	Hadamard products involving nonnegative matrices and M -matrices 356
Chapter 6		Matrices and functions 382
	6.0	Introduction 382
	6.1	Polynomial matrix functions and interpolation 383
	6.2	Nonpolynomial matrix functions 407
	6.3	Hadamard matrix functions 449
	6.4	Square roots, logarithms, nonlinear matrix equations 459
	6.5	Matrices of functions 490
	6.6	A chain rule for functions of a matrix 520
		Hints for problems 561
		References 584
		Notation 590
		Index 595

Preface

This volume is a sequel to the previously published *Matrix Analysis* and includes development of further topics that support applications of matrix theory. We refer the reader to the preface of the prior volume for many general comments that apply here also. We adopt the notation and referencing conventions of that volume and make specific reference to it [HJ] as needed.

Matrix Analysis developed the topics of broadest utility in the connection of matrix theory to other subjects and for modern research in the subject. The current volume develops a further set of slightly more specialized topics in the same spirit. These are: the field of values (or classical numerical range), matrix stability and inertia (including M -matrices), singular values and associated inequalities, matrix equations and Kronecker products, Hadamard (or entrywise) products of matrices, and several ways in which matrices and functions interact. Each of these topics is an area of active current research, and several of them do not yet enjoy a broad exposition elsewhere.

Though this book should serve as a reference for these topics, the exposition is designed for use in an advanced course. Chapters include motivational background, discussion, relations to other topics, and literature references. Most sections include exercises in the development as well as many problems that reinforce or extend the subject under discussion. There are, of course, other matrix analysis topics not developed here that warrant attention. Some of these already enjoy useful expositions; for example, totally positive matrices are discussed in [And] and [Kar].

We have included many exercises and over 650 problems because we feel they are essential to the development of an understanding of the subject and its implications. The exercises occur throughout the text as part of the

viii **Preface**

development of each section; they are generally elementary and of immediate use in understanding the concepts. We recommend that the reader work at least a broad selection of these. Problems are listed (in no particular order) at the end of sections; they cover a range of difficulties and types (from theoretical to computational) and they may extend the topic, develop special aspects, or suggest alternate proofs of major ideas. In order to enhance the utility of the book as a reference, many problems have hints; these are collected in a separate section following Chapter 6. The results of some problems are referred to in other problems or in the text itself. We cannot overemphasize the importance of the reader's active involvement in carrying out the exercises and solving problems.

As in the prior volume, a broad list of related books and major surveys is given prior to the index, and references to this list are given via mnemonic code in square brackets. Readers may find the reference list of independent utility.

We appreciate the assistance of our colleagues and students who have offered helpful suggestions or commented on the manuscripts that preceded publication of this volume. They include M. Bakonyi, W. Barrett, O. Chan, C. Cullen, M. Cusick, J. Dietrich, S. H. Friedberg, S. Gabriel, F. Hall, C.-K. Li, M. Lundquist, R. Mathias, D. Merino, R. Merris, P. Nysten, A. Sourour, G. W. Stewart, R. C. Thompson, P. van Dooren, and E. M. E. Wermuth.

The authors wish to maintain the utility of this volume to the community and welcome communication from readers of errors or omissions that they find. Such communications will be rewarded with a current copy of all known errata.

R. A. H.
C. R. J.

Preface to the Second Printing

We have corrected all known errata in the first printing, polished the exposition of a few points, noted the resolution of several conjectures, and added some items to the notation list and index. It is a pleasure to acknowledge helpful comments from our colleagues T. Ando, R. Bhatia, S. Friedberg, D. Jespersen, B. Kroschel, I. Lewkowicz, C.-K. Li, R. Loewy, J. Miao, and F. Uhlig.