INFRARED ASTRONOMY

IV Canary Islands Winter School of Astrophysics

Edited by

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1 INFRARED ASTRONOMY AND STAR FORMATION

In a report of the current status of the observational studies of star formation published in 1982, Wynn-Williams introduced the subject by boldly stating "*Protostars are the Holy Grail of infrared astronomy*", one of the most (ab)used astronomical quotations ever. At the time of the review, searches had been made at IR wavelengths towards a restricted sample of objects, guided by the theoretical expectation that protostars would undergo a phase of high IR luminosity during the main accretion phase (Larson 1969). The aim of these studies, and of those that followed, was to get an unambiguous example of Spitzer's definition (1948) of a protostar as "*an isolated interstellar cloud undergoing inexorable gravitational contraction to form a single star*".

This definition appears now rather restrictive, since observations have shown that the structure of star forming regions is highly complex, rarely revealing well isolated, noninteracting interstellar clouds. In addition, observations in the near infrared at high spatial resolution have shown that many young stars are not single, but do have companions (Zinnecker & Wilking 1992). However, despite great effort, resulting from improvements in instrumentation and progress in the theoretical models, the same discouraging conclusion reached by Winn-Williams in 1982 still holds true: no conclusive identification of a genuine protostar has yet been made. Nevertheless, the motivation for continuing the search using the IR band remains still unquestionable.

In these lectures I will present an overview of the main properties of the star formation process with a special emphasis on stars of low- and intermediate-mass, for which observations are the most detailed and a consistent theoretical framework has been developed and tested. The first Section describes the infrared properties of star forming regions, and the modes of star formation within giant molecular clouds. The second lecture is devoted to the analysis of the gravitational collapse of dense molecular cores and the protostellar accretion phase. The third lecture reviews the theoretical foundation of pre–main-sequence evolution, and describes the circumstellar environment of young stars. The last lecture focuses on the properties of young embedded stellar clusters, and the derivation of infrared and bolometric luminosity functions.

1.1 The Quest for Protostars

By definition, a protostar is an object that derives its energy from the conversion of infall kinetic energy to radiation, and that has not yet reached hydrostatic equilibrium. The photons produced at the accretion shock, either bounding the protostellar core or a circumstellar disk, are effectively absorbed by the large amount of residual circumstellar matter and reemitted at far-infrared wavelengths, typically longward of 30μm. In 1982, the list of candidates consisted of about 30 infrared sources, all of them with bolometric luminosities in excess of 10^3 $L_\odot$. Such a high luminosity implies the presence of embedded sources with masses greater than ~ 3 $M_\odot$, presumably fully formed stars of the early spectral types. (Yorke & Shustov 1981). Since the distribution of stellar masses, the initial mass function (IMF), peaks at values less than 1 $M_\odot$ (Salpeter 1955; Scalo 1986), it is clear that all the searches carried out until then were unable to probe the most common byproduct of star formation, solar type stars.

The fact that none of the candidate protostars satisfied Spitzer's definition derived not from a shortage of candidates (albeit the number was indeed small), but from the lack of knowledge of their evolutionary status. In order to recognize a protostar, two requirements must be fulfilled: first, the infrared continuum spectrum should have the expected characteristics; second, the infalling gas should show evidence of dynamical collapse in the line profile, as indicated by an inverse P-Cygni profile with the redshifted absorption appearing against either a continuum source or the emission line. A decade ago these constraints posed unsurmountable limits to succeed in the quest: it is the progress both in the observational capabilities, in the infrared as well as in the submillimeter and millimeter wavebands, and in the theoretical understanding of the basic physical processes that have made the problem of star formation and the quest for protostars much less frustrating ten years later. The main events that have marked this period of time are schematically listed below. Of course, this list is by no means complete, but it is merely intended as a guideline for the following discussion.

Dense Molecular Cores: it has been recognized that high density molecular cloud cores, as traced by NH_3, CS and $C^{18}O$ molecules, represent the initial conditions for the star formation process, mainly of low- and intermediate-mass stars (Myers & Benson 1983).

IRAS: the satellite has provided the first complete map of most of the sky ($\sim$ 95%). The Point Source Catalogue (PSC), with its approximately 250000 entries, has offered the possibility of approaching the statistical aspects of star formation in a systematic and unbiased way (Beichman *et al.* 1984).

Winds & Outflows: it has been demonstrated that, opposite to the theoretical expectation, the main kinematic signature of star forming clouds is *outflow* rather than *infall*. The earliest evolutionary phases of *all* stars are marked by the occurrence of ejection of matter in the form of powerful winds, as traced by P-Cygni profiles, H-H objects, maser emission, and optical jets (Lada C. 1985).

IR Classification of YSOs: an empirical scheme based on the shape of the broad-band (1-100μm) spectral energy distribution has been devised to classify young stellar objects (Lada C. 1987), and to infer their evolutionary state (Shu, Adams & Lizano 1987).

NIR Array Cameras: a veritable revolution in instrumentation that allows to image large areas of star forming regions, with both high spatial resolution and sensitivity. The IR cameras have revealed the existence of many visually obscured stellar clusters in giant molecular clouds; probed the low-mass stellar population associated with HII regions, thus overcoming the basic limitation of previous surveys, and shed light on the stellar Luminosity Function at birth (Gatley *et al.* 1991; see also chapter by I. McLean).

1.2 IRAS and Star Formation

The IRAS survey, based on four wavebands centred at 12, 25, 60 and 100 μm, has produced a catalogue of point sources essentially complete, at the 10σ level, to 0.7, 0.65, 0.8, and 3.0 Jy, respectively. This enormous database has been used to select, extract and analyse sample of sources solely on infrared criteria. For studies of star formation, the classification of IRAS sources using their spectral energy distributions (SED) has proved extremely useful to separate young stellar objects in different evolutionary states. Also, the use of color-color plots, based on the measured flux densities, has allowed the definition of selection criteria to distinguish different classes of objects. The results are summarized in Table I. The location of young stellar objects in selected color-color diagrams is illustrated in Figure 1. An analysis of the problems encountered in selecting highly complete and not contaminated samples of young stellar objects in the IRAS Point Source Catalogue can be found in Prusti *et al.* (1991).

Dense Cores & Protostars: The dense cores observed in ammonia by Myers and Benson (1983) have been used as prime targets to search for embedded sources of low luminosity. In fact, IRAS had the adequate sensitivity at long wavelengths to detect all low luminosity objects in nearby star forming regions (Beichman *et al.* 1986). For example, the IRAS survey samples the entire stellar population for $L_* \gtrsim 0.5\ L_\odot$ in the Taurus-Auriga complex (Kenyon *et al.*

Class of Object	Log (F_{25}/F_{12})	Log (F_{60}/F_{25})	Log (F_{100}/F_{60})	Log (F_{60}/F_{12})	Ref.
Dense Cores	0.4 – 1.0	0.4 – 1.3	0.1 – 0.7	—	1
Maser H_2O	0.5 – 1.1	0.4 – 1.7	−0.1 – 1.5	—	2
UC HII regions	> 0.6	—	—	> 1.3	3
T Tauri stars	0.03 – 0.58	−0.26 – 0.51	0.0 – 0.4	—	4
Herbig Ae/Be	−0.1 – 0.6	−0.2 – 1.0	−0.2 – 0.5	—	5

1 Emerson (1987); 2 Wouterloot & Walmsley (1986); 3 Wood & Churchwell (1989); 4 Harris *et al.* (1988); 5 Berrilli *et al.* (1990)

Table I: *IRAS selection criteria for young stellar objects*

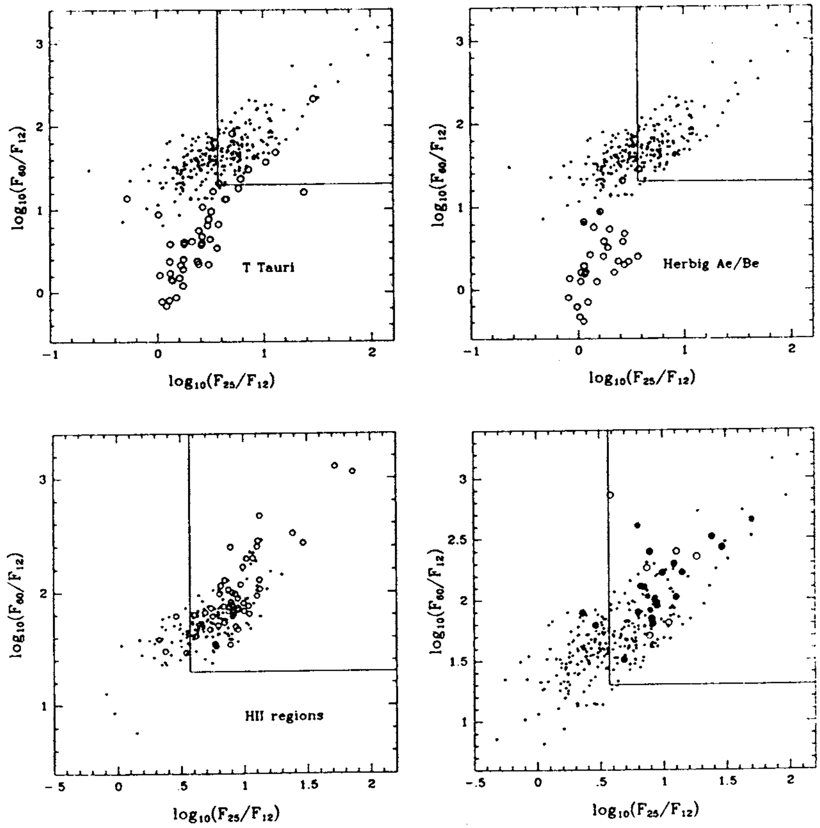

Figure 1: *Color-color diagrams of various classes of objects. Empty circles refer to T Tauri stars, Herbig Ae/Be stars, and H*II *regions. The small filled circles show the location of IRAS sources associated with dense molecular clouds and ultracompct H*II *regions. The solid lines delimit the colors for embedded massive stars proposed by Wood & Churchwell (1989).*

1990), and in Ophiucus (Wilking 1989). At the distance of Orion (~450 pc), the nearest giant molecular complex, the census is limited to sources with $L_* \gtrsim 6$ $L_\odot$ (Strom *et al.* 1989a). Thus, the stellar populations hosted in these regions cannot be directly compared on the basis of the IRAS data alone, but candidate protostars can be searched systematically in the closer clouds.

The association of the IRAS point sources with the dense cores has been secured in approximately half of the known cores, and typically one to few stars are found per core (Benson & Myers 1989). The remaining half of the cores are thought to be in the process of forming stars. The nature of the embedded sources has been determined by follow-up studies at optical and NIR wavelengths. In most cases the objects are young stars (T Tauri-like), still surrounded by large amounts of circumstellar matter. The rest of the sources remain optically invisible, with an estimated extinction in excess of 30 mag: they represent the best candidates of protostars (Myers *et al.* 1987; Ohashi *et al.* 1991). The knowledge of the relative proportion of the two classes of objects in several star forming regions has given an indirect estimate of the duration of the embedded phase, $\tau_{\rm emb} \sim 10^5$ yr. If the typical source has a solar mass, $\tau_{\rm emb}$ would imply an average mass accretion rate of $\dot{M}_{\rm acc} \sim 10^{-5} M_\odot {\rm yr}^{-1}$, in agreement with theoretical predictions (Stahler, Shu & Taam 1980). However, the accretion rate derived from the measured bolometric luminosity can be substantially smaller, $\dot{M}_{\rm acc} \sim 10^{-7}$ $M_\odot {\rm yr}^{-1}$ (Myers *et al.* 1987; Kenyon *et al.* 1990), suggesting that very few sources can be catched during the main accretion phase.

Massive Stars & HII regions: The formation of massive stars takes also place in dense cores within molecular cloud complexes. A powerful two color selection criterion has been developed by Wood & Churchwell (1989) to identify massive stars still embedded inside molecular clouds. These sources are characterized by the largest flux densities at 100 μm, implying that the emitting dust is quite cool (T_d ~30 K); their spectra peak at $\sim 100\mu$m and the shape does not show an appreciable variation from source to source. As shown in the color-color plots of Fig. 1, the population of embedded massive stars, and their associated ultracompact HII regions, occupies a limited and well defined portion of the diagram, away from other entries in the PSC. Their distinctive colors have been used to estimate the total number and distribution of embedded massive stars in the Galaxy, the timescale of the embedded phase (typically ~10-20% of the main-sequence lifetime), and the current rate of massive star formation ($\sim 3 \times 10^{-3}$ O stars yr^{-1}). Follow-up studies in molecular lines, NH_3, CS, ^{13}CO and H_2O maser transition (Churchwell *et al.* 1990; Palla *et al.* 1991; Cesaroni *et al.* 1992) have largely confirmed the association between the IRAS sources, dense molecular gas and UC HII regions.

T Tauri and Herbig Ae/Be stars: A large fraction of the catalogued optically visible, pre–main-sequence (PMS) stars have an IRAS association. The studies by Harris *et al.* (1988) and Weintraub (1990) of a sample of T Tauri stars in Taurus have shown that their colors are tightly confined by the bounds given in Table 1. The higher mass counterparts, the Herbig Ae/Be stars, have very similar color, although the scatter is larger (Berrilli *et al.* 1990). This is somewhat surprising, given the different luminosities of the two classes of stars. In both cases, the color corresponds to uncorrected temperatures of few hundred Kelvin. The analysis of the spectral energy distribution has provided the first indirect evidence of the presence of circumstellar disks around young stars (Rucinski 1985). We will return in Section 3 to the IR properties of the circumstellar matter associated with PMS stars.

1.3 The IR classification scheme

Studies of the *shape* of the spectral energy distribution (SED) of young stellar objects (YSOs) have proved very useful to determine their nature and evolutionary state. Since the emitted spectrum from a YSO depends on the distribution and physical properties (density, temperature and composition) of the surrounding dust and gas, it is natural to expect a dependence of its shape on the evolutionary state of the source. A protostellar object deeply embedded in the parent cloud should have an infrared signature markedly different than that of a mature PMS star, where most of the material has been accumulated onto the central object. An empirical classification scheme has been developed, based on the slope of the SED longward of 2.2 μm (Lada C. 1987; Wilking 1989). For each SED, a spectral index $\alpha \equiv -dlog(\nu F_\nu)/dlog\nu$, with F_ν the flux density at wavelength λ, is computed between 2.2 and 25 μm, and the resulting morphological classes are shown in Figure 2.

Class I sources have $\alpha > 0$, indicating a rise in the SED that often continues even at longer wavelengths. The IR excess is very conspicuous and the SED is much broader than that of a single temperature blackbody function. *Class II* sources have $-2 < \alpha < 0$. Their SEDs fall towards longer wavelenghts, but are still broad, due to a significant amount of circumstellar dust. Finally, *Class III* sources have $\alpha < -2$, and the SED resembles that of a normal, reddened stellar photosphere. As it always is the case in astronomy, the classification represents a powerful tool, but the scheme is not so rigid. A variety of YSOs have been found with intermediate properties, and additional subclasses (labeled "D") have been introduced to account for double-peaked SEDs.

The interpretation in terms of an evolutionary sequence is sketched in the

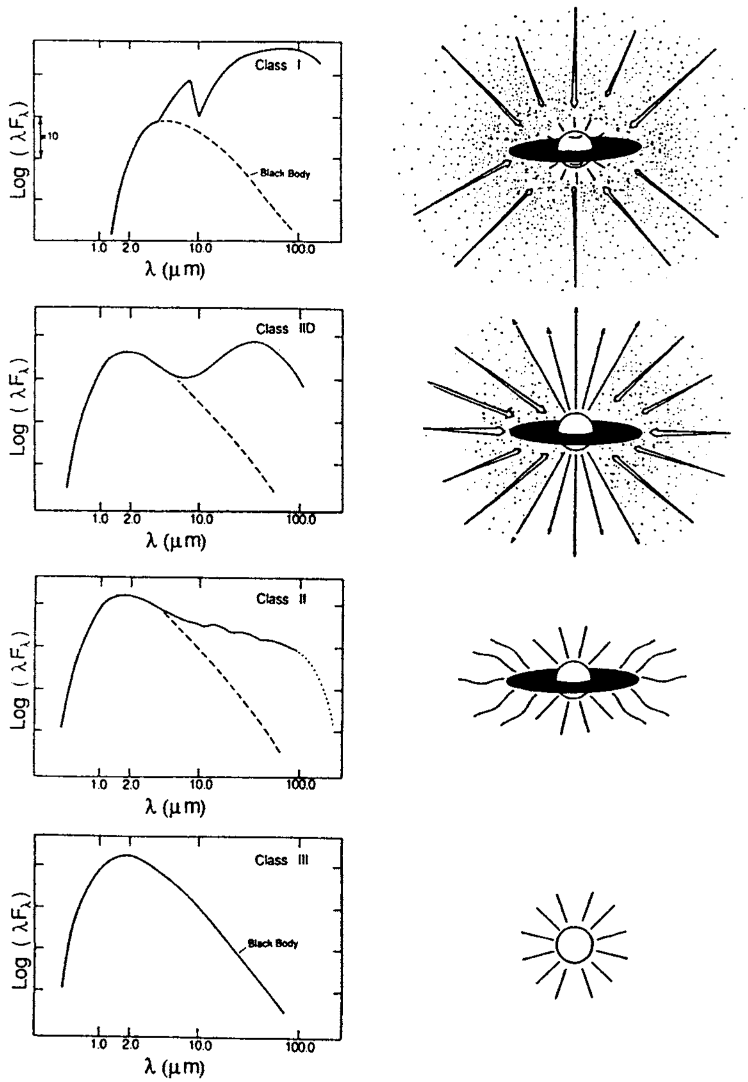

Figure 2: *Classification scheme (left) and evolutionary sequence (right) for young stellar objects (adapted from Lada C. 1991 and Shu et al. 1987).*

right panel of Fig. 2 (Shu *et al.* 1987). *Class I* sources are thought to represent accreting protostars, surrounded by luminous disks, with extent of $\sim$ 10–100AU, and by infalling, spherically symmetric envelopes with typical sizes of $\sim 10^4$AU. The stage of partial clearing of the obscuring matter where the central star begins to be optically visible, due to the action of a powerful stellar wind, corresponds to Class IID. A *Class II* source results from the final disruption of the envelope, and corresponds to a classical T Tauri or Herbig Ae/Be star surrounded by spatially thin, optically thick circumstellar disk of radius $\sim$ 100AU. Finally, the absence of the IR excess implies the disappearance of the circumstellar structures, disks and envelopes, and the approach to the conditions of a normal main sequence star (*Class III*). The evolutionary scenario in four stages (the initial stage of core formation has been omitted in this discussion; cf. Shu *et al.* 1993) has been widely used to interpret the observations towards individual star forming regions, and to elucidate the nature of Class I sources, the elusive protostars.

The reason for the lack of success in the identification of a genuine example of a protostar is now becoming more clear. It has been suggested that Class I sources are in fact more evolved objects than true protostars, having already assembled most of the final stellar mass (André *et al.* 1993). Support to this view comes from studies at millimeter and sub-millimeter wavelengths of a sample of deeply embedded sources. Continuum observations at these wavelengths probe the density and temperature distribution of the circumstellar material, and allow a direct estimate of the mass in the form of dust and of the total luminosity of the source (Mezger *et al.* 1991; Zinnecker et al. 1992). By definition, Class I sources should be surrounded by large amounts of dust, but the observations have surprisingly shown that this is not the case: the mass in the envelopes does not differ significantly from that measured around more evolved PMS stars (Beckwith *et al.* 1990). Once again, the mistery remains. Perhaps, the final answer will come from the detailed study of a class of sources so deeply hidden within the parent cloud that they cannot be detected *even* at infrared wavelengths. An example of the SED emitted from such a source, VLA 1623, is shown in Figure 3. Unlike Class I objects, the distinctive feature here is that the SED is well fitted by a single temperature blackbody function. André *et al.* (1993) have argued that VLA 1623 is the prototype of an entirely new class of (proto)stellar objects, called *Class 0*, and proposed a new scheme of age ordering of YSOs based on their sub-mm luminosities. The interesting aspect of this source is that, despite its extreme youth, it drives a well developed bipolar CO outflow. The final proof that this, or similar sources are indeed the long sought for protostars must however await the appropriate kinematic evidence of gravitational infall.

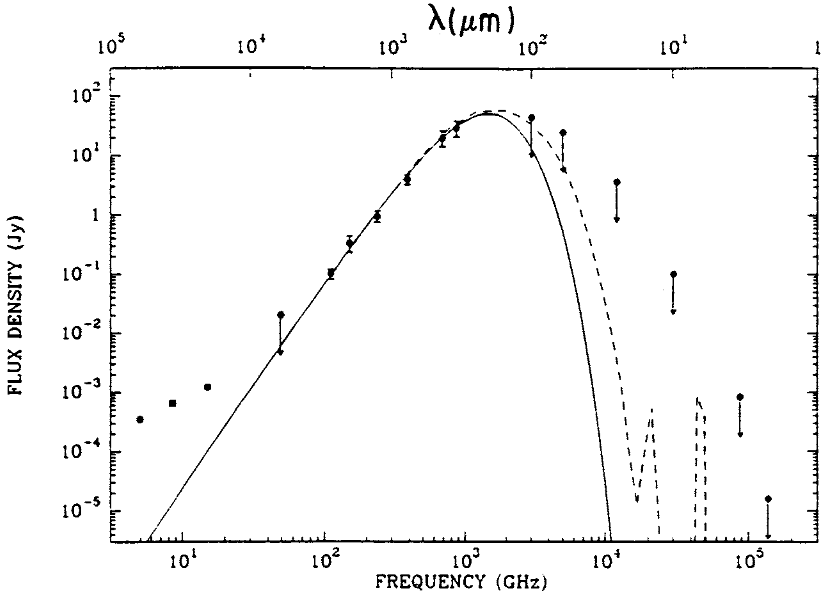

Figure 3: *The spectral energy distribution of VLA 1623, the prototype of a class of YSOs that only show at wavelengths longer than 10μm. The solid curve is a fit with a single temperature blackbody function. The dashed curve represents radiative transfer modelling (from André et al. 1993).*

1.4 Modes and distribution of star formation

The knowledge of the distribution of star forming regions within giant molecular clouds (GMCs) provides a clue to understanding the star formation mechanisms. Schematically, two extreme situations have been envisaged as being the dominant modes of star formation: a *spontaneous* mode, in which the formation process occurs without external influence, and an externally *stimulated* mode. Which of the two is prevalent in molecular clouds, however, is not well understood. The advent of NIR array cameras has offered the appropriate tool to properly address this question. Let us consider each of these schemes in some detail.

1.4.1 Stimulated Star Formation

The variety of physical agents that can induce star formation under external influences has been thoroughly presented by Elmegreen (1992). The basic idea is that external forces, represented by the action of stellar winds, ionization fronts, supernova explosions, result in a compression of pre-exisiting clouds, altering their stable configuration and making them suitable for gravitational collapse. The propagation of shock fronts rises the gas density and enhances the local rate of turbulent, magnetic, or rotational energy dissipation, leaving the gas in a state close to virialization. Clouds can then collapse, and, because of the higher initial density, they do so in a faster time scale. On the other hand, the presence of strong shocks also influences the gas bulk motions, enhancing the kinetic energy of the clouds. Their overall stability against collapse is thus favored, and in some cases, shocks and ionization fronts can even induce the evaporation of a large fraction of the original interstellar material. Numerical simulations of the interaction of ionized radiation and expanding HII regions with globules have shown the twofold effects on the interstellar condensations: an increase both in the *mass loss* rate, by evaporation of the neutral matter from the surfaces, and in the *mass accretion* rate, by compression of the gas to higher densities (Klein *et al.* 1985; Bertoldi & McKee 1990).

From an observational viewpoint, the outcome of the triggered mode is that star formation should take place predominantly near sites of previous generations of massive stars, and not throughout a GMC. The best example of such a situation is provided by the linear sequences of stellar subgroups in OB associations (Blaauw 1964; 1991). But stimulated star formation can also affect the large scale distribution of the interstellar gas, creating holes, shells and bubbles (Tenorio-Tagle & Bodenheimer 1988). On an even larger scale, strong dynamical effects can induce the formation of galactic fountains and shape the structure of a galaxy as a whole (Franco 1992). More convincing examples of the *small scale* range of induced star formation are the bright rimmed globules observed by Sugitani *et al.* (1991) and Tauber *et al.* (1993). These globules have the same physical characteristics of the quiescent dense cores, but there is evidence that the embedded IRAS point sources could represent stars of intermediate mass. Thus, more massive stars are produced by this process than would otherwise if the clouds were in isolation.

1.4.2 Spontaneous Star Formation

According to this idea, the evolution of an individual star forming cloud is determined solely by the balance of the destabilizing effects of self-gravity and the restoring forces due to thermal pressure, plus turbulence, rotation and magnetic fields. Each of these energy reservoirs has to be dissipated *before* gravitational collapse can occur. Thus, the typical timescale for star formation is not governed by the gravitational free-fall time, that at the typical densities of molecular clouds is of order 10^{5-6} yr, but by the dissipation time, that can amount to $\sim 10^7$ yr (e.g. Elmegreen 1991). The physical processes that govern the spontaneous mode are conducive of a situation where a dense core, of the type discussed by

Myers and collaborators, gradually builds up inside the larger molecular cloud, presumably via the action of ambipolar diffusion (Lizano & Shu 1989). This stage then corresponds to the first phase of the evolutionary scenario outlined above, and represents the initial conditions for the actual phase of gravitational collapse.

Observationally, the spontaneous mode predicts a more or less uniform spatial distribution of stars throughout a GMC complex. Examples of the location of young stars in several regions were discussed by Larson (1982), who noted that while in Taurus most emission line stars are scattered through the cloud (and similarly in the streamers of Ophiucus), the opposite is true in Orion: the majority of the stars is in large clusters, with high degree of central condensations. However, the data used by Larson were representative only of limited portions of the molecular clouds in Orion, and definite conclusions about the nature of star formation could not be drawn. At present, the situation has greatly improved, thanks to the developments of IR array cameras and millimeter wave telescopes, and a more definite picture is emerging.

Clustered Mode: In order to obtain a complete census of both the stellar population (embedded and optically revealed stars) *and* the dense molecular gas, large-scale mapping over extended areas must be carried out. The results of recent unbiased, systematic surveys in individual molecular clouds in the Orion complex have revealed that the clustered mode is indeed prevalent (Lada E. 1992; Lada E. *et al.* 1993; Tatematsu *et al.* 1993; Umemoto *et al.* 1993). Figure 4 shows the distribution of stellar clusters and dense gas in L1630. The impressive findings of the survey in this cloud indicate that almost all the sources detected at 2.2μm and associated with the molecular cloud are contained in four clusters only; that these clusters occupy a very small fraction (~18%) of the surveyed area and are coincident with the most massive cores revealed in CS. The star formation efficiency can be quite high within these cores (~30-40%), although overall is rather modest (~3-4%), in accordance with previous estimates.

Thus, it appears that the clustered mode of star formation prevails over the distributed mode even in the case of low-mass stars. A conclusion supported also by the discovery through infrared camera work of hundreds of such stars associated with luminous OB stars in, e.g., S106, M17, W3OH (see last Section). If Orion represents an average GMC, then Lada E. *et al.* (1993) suggest that the clustered mode should account for the formation of the bulk of the stars in our Galaxy, independent of their mass. The problem of star formation is thus shifted to the question of how massive cores form in the first place, and produce rich clusters of stars.

Isolated mode: This subsection cannot be complete without a mention to the

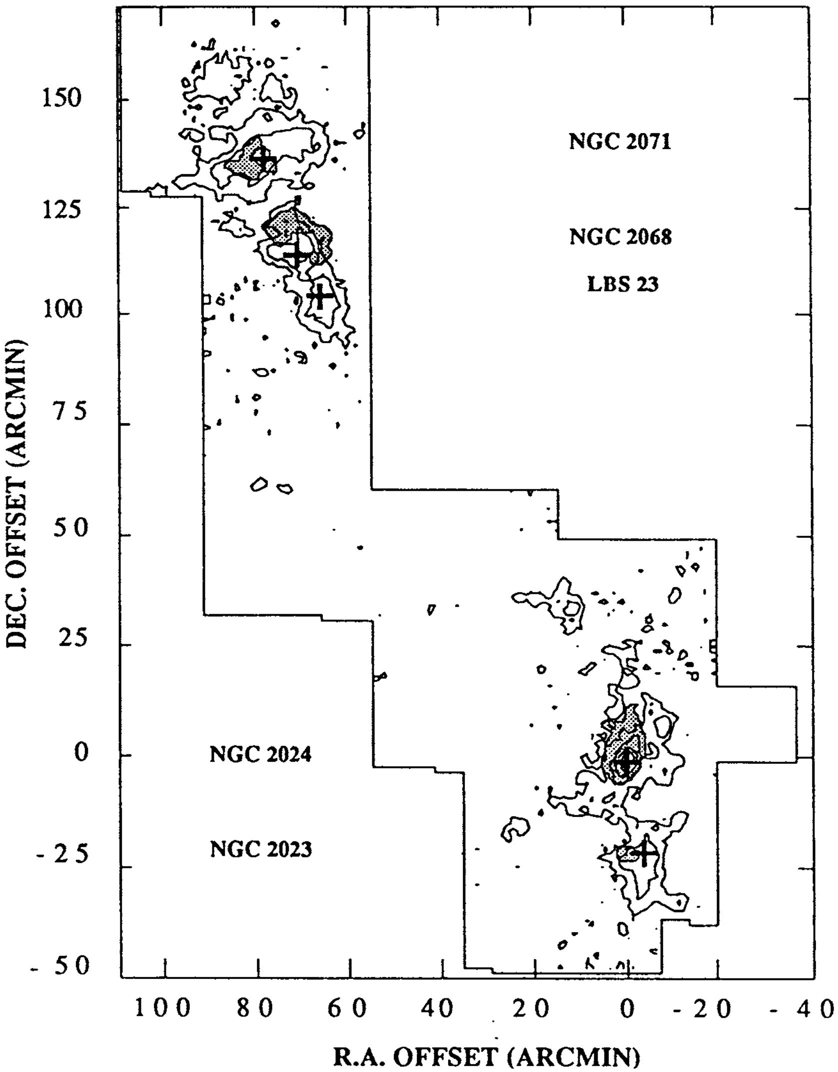

Figure 4: *Maps of the embedded stellar cluster (shaded regions) and dense gas traced by CS emission (intensity contours) in L1630 in Orion. Crosses mark the positions of the most massive CS cores. The survey consists of 3000 1'x1' fields imaged at 2.2μm, down to a completeness limit of 13 mag, and 13000 points with 1' spacing observed in CS (from Lada E. 1992).*

best examples of isolated cases of star formation, the so-called Bok globules (Bok & Reilly 1947). These small (~0.1 pc), cold (~10 K), isolated dark clouds have long been suspected to be sites of low-mass star formation, but until recently direct evidence has been missing. In fact, detailed analysis have emphasized their overall dynamical stability against gravitational collapse, with possibly very few exceptions (e.g. B5, B335). Infrared studies have now confirmed Bok's original suggestion. It has been shown that a large fraction of them hosts far-infrared point sources (Yun & Clemens 1990; indeed, they argue that *all* Bok globules harbour a newly formed star), and molecular outflows (Parker *et al.* 1991; Yun & Clemens 1992). The high frequency of the outflows, their estimated duration ($\sim 10^5$yr) and energetics (total kinetic energy~gravitational binding energy) strongly indicate their major role in driving the early evolution of low-mass young stars. The next Section describes in detail the physical processes that govern the formation of stars under the conditions of this *isolated mode*.

2 ISOLATED STAR FORMATION

The formation of a protostar from the quiescent conditions typical of dense cores occurs through the gradual accumulation of the interstellar gas onto an accreting core. This process requires a large decrease in gravitational potential energy. A large fraction of this energy is radiated away in radial or disk accretion shocks that form as a result of the abrupt change in the velocity of the freely-falling gas. If no net energy is absorbed by the circumstellar material, the resulting luminosity is given to a good approximation by

$$L_{\rm proto} = \frac{GM_*\dot{M}_{\rm acc}}{R_*} , \tag{1}$$

where M_* and R_* are the instantaneous mass and radius of the protostellar core, and $\dot{M}_{\rm acc} = dM_*/dt$ the mass accretion rate. Thus, estimates of the luminosity emitted during this phase rely on the knowledge of two fundamental quantities: *the mass accretion rate*, and the *mass-radius relation*. The former is determined by the dynamics of the gravitational collapse, while the relation between the mass and the radius are established by processes occuring in the protostellar interior. The radiation produced at the shock is absorbed, reradiated and thermalized in the optically thick dusty infalling envelope. Most of the observable radiation is emitted at mid- and far-infrared wavelengths. The exact shape of the emergent spectrum depends on the density and temperature distribution of the dust component, which are set by the dynamics of collapse. This section describes how current models of protostellar formation and evolution have constrained both the value of the mass accretion rate, and the structural properties of the central core. Some relevant observational tests will be presented at the end.

2.1 Protostellar collapse: the mass accretion rate

An estimate of $\dot{M}_{\rm acc}$ requires the solution of the dynamical collapse problem. In the idealized case of the collapse of a marginally unstable cloud, such a solution has been found semi-analitically by Shu (1977). In this theory, the rate at which a protostar is built up is

$$\dot{M}_{\rm acc} = \alpha \, \frac{a_{\rm eff}^3}{G} \, . \tag{2}$$

In eq. (2), $a_{\rm eff}$ is the effective isothermal sound speed, G the gravitational constant, and α a constant of order unity, If the cloud is supported only by thermal pressure, the expression for the sound speed is simply $a_{\rm eff} = \sqrt{(KT/m)}$, where