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0521461979 - Topics in the Constructive Theory of Countable Markov Chains

G. Fayolle, V. A. Malyshev and M. V. Menshikov

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