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0521460816 - Basic Abstract Algebra, Second Edition - P. B. Bhattacharya, S. K. Jain and S. R. Nagpaul

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