BASIC ABSTRACT ALGEBRA

Second edition

Basic abstract algebra

Second edition

P. B. BHATTACHARYA
Formerly, University of Delhi

S. K. JAIN
Ohio University

S. R. NAGPAUL
St. Stephen's College, Delhi

CAMBRIDGE
UNIVERSITY PRESS

Published by the Press Syndicate of the University of Cambridge
The Pitt Building, Trumpington Street, Cambridge CB2 1RP
40 West 20th Street, New York, NY 10011-4211, USA
10 Stamford Road, Oakleigh, Melbourne 3166, Australia

First edition published 1986
Second edition published 1994
Reprinted 1995

Library of Congress Cataloging-in-Publication Data is available.

A catalogue record for this book is available from the British Library.

ISBN 0-521-46081-6 hardback
ISBN 0-521-46629-6 paperback

Transferred to digital printing 2001

For
PARVESH JAIN
To whom we owe more than we can possibly expres

Contents

Preface to the second edition

The following are the main features of the second edition.

More than 150 new problems and examples have been added. The new problems include several that relate abstract concepts to concrete situations. Among others, we present applications of G-sets, the division algorithm and greatest common divisors in a given euclidean domain. In particular, we should mention the combinatorial applications of the Burnside theorem to real-life problems. A proof for the constructibility of a regular n-gon has been included in Chapter 18.

We have included a recent elegant and elementary proof, due to Osofsky, of the celebrated Noether–Lasker theorem.

Chapter 22 on tensor products with an introduction to categories and functors is a new addition to Part IV. This chapter provides basic results on tensor products that are useful and important in present-day mathematics.

We are pleased to thank all of the professors and students in the many universities who used this textbook during the past seven years and contributed their useful feedback. In particular, we would like to thank Sergio R. Lopez-Permouth for his help during the time when the revised edition was being prepared. Finally, we would like to acknowledge the staff of Cambridge University Press for their help in bringing out this second edition so efficiently.

P. B. Bhattacharya
S. K. Jain
S. R. Nagpaul

Preface to the first edition

This book is intended for seniors and beginning graduate students. It is self-contained and covers the topics usually taught at this level.

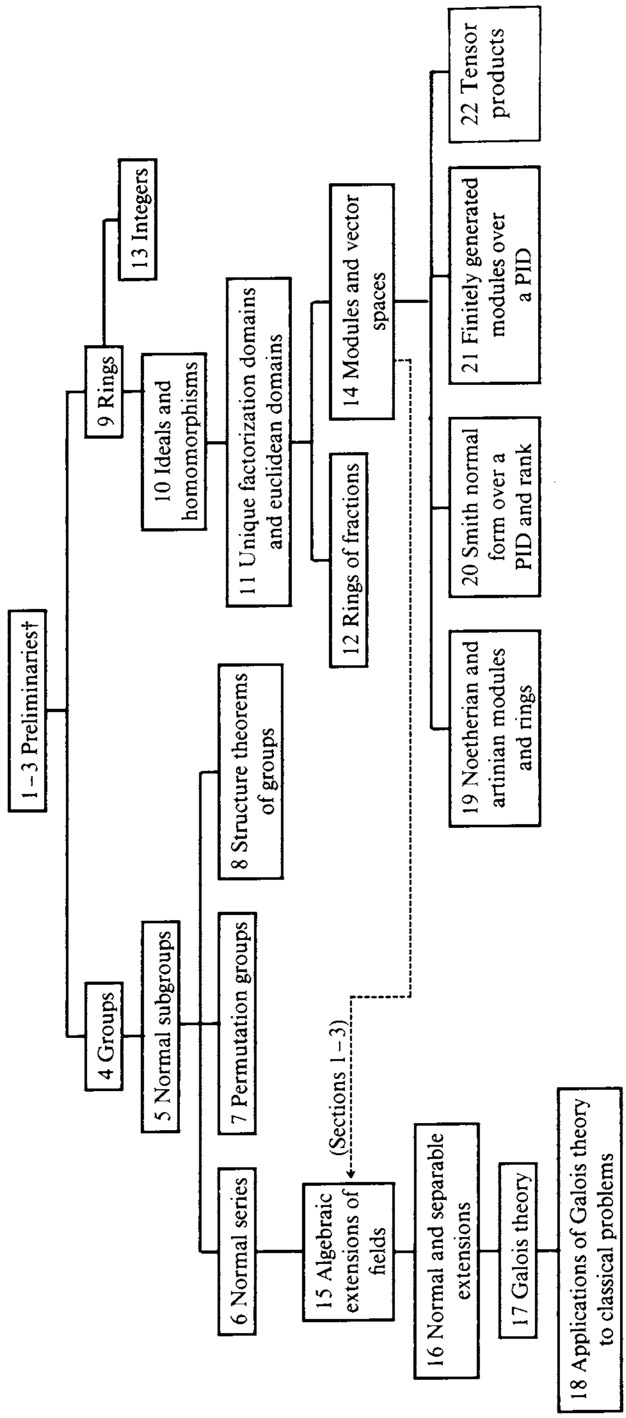
The book is divided into five parts (see diagram). Part I (Chapters 1–3) is a prerequisite for the rest of the book. It contains an informal introduction to sets, number systems, matrices, and determinants. Results proved in Chapter 1 include the Schröder–Bernstein theorem and the cardinality of the set of real numbers. In Chapter 2, starting from the well-ordering principle of natural numbers, some important algebraic properties of integers have been proved. Chapter 3 deals with matrices and determinants. It is expected that students would already be familiar with most of the material in Part I before reaching their senior year. Therefore, it can be completed rapidly, skipped altogether, or simply referred to as necessary.

Part II (Chapters 4–8) deals with groups. Chapters 4 and 5 provide a foundation in the basic concepts in groups, including G-sets and their applications. Normal series, solvable groups, and the Jordan–Hölder theorem are given in Chapter 6. The simplicity of the alternating group A_n and the nonsolvability of S_n, $n > 4$, are proved in Chapter 7. Chapter 8 contains the theorem on the decomposition of a finitely generated abelian group as a direct sum of cyclic groups, and the Sylow theorems. The invariants of a finite abelian group and the structure of groups of orders p^2, pq, where p, q are primes, are given as applications.

Part III (Chapters 9–14) deals with rings and modules. Chapters 9–11 cover the basic concepts of rings, illustrated by numerous examples, including prime ideals, maximal ideals, UFD, PID, and so forth. Chapter 12 deals with the ring of fractions of a commutative ring with respect to a multiplicative set. Chapter 13 contains a systematic development of

Cambridge University Press
0521460816 - Basic Abstract Algebra, Second Edition - P. B. Bhattacharya, S. K. Jain and S. R. Nagpaul
Frontmatter/Prelims
More information

TABLE OF INTERDEPENDENCE OF CHAPTERS



† To be read as and when needed.

integers, starting from Peano's axioms. Chapter 14 is an introduction to modules and vector spaces. Topics discussed include completely reducible modules, free modules, and rank.

Part IV (Chapters 15–18) is concerned with field theory. Chapters 15 and 16 contain the usual material on algebraic extensions, including existence and uniqueness of algebraic closure, and normal and separable extensions. Chapter 17 gives the fundamental theorem of Galois theory and its application to the fundamental theorem of algebra. Chapter 18 gives applications of Galois theory to some classical problems in algebra and geometry.

Part V (Chapters 19–21) covers some additional topics not usually taught at the undergraduate level. Chapter 19 deals with modules with chain conditions leading to the Wedderburn–Artin theorem for semisimple artinian rings. Chapter 20 deals with the rank of a matrix over a PID through Smith normal form. Chapter 21 gives the structure of a finitely generated module over a PID and its applications to linear algebra.

Parts II and III are almost independent and may be studied in any order. Part IV requires a knowledge of portions to Parts II and III. It can be studied after acquiring a basic knowledge of groups, rings, and vector spaces. The precise dependence of Part IV on the rest of the book can be found from the table of interdependence of chapters.

The book can be used for a one-year course on abstract algerba. The material presented here is in fact somewhat more than most instructors would normally teach. Therefore, it provides flexibility in selection of the topics to be taught. A two-quarter course in abstract algebra may cover the following: groups – Chapters 4, 5, and 7 (Section 1) and 8; rings – Chapters 9, 10, 11, and 14 (Sections 1–3); field theory – Chapters 15, 16, and 18 (Section 5). A two-semester course in abstract algebra can cover all of the material in Parts II, III, and IV.

Numerous examples have been worked out throughout the book to illustrate the concepts and to show the techniques of solving problems. There are also many challenging problems for the talented student. We have also provided solutions to the odd-numbered problems at the end of the book. We hope these will be used by students mostly for comparison with their own solutions.

Numbering of theorems, lemmas, and examples is done afresh in each chapter by section. If reference is made to a result occurring in a previous chapter, then only the chapter number is mentioned alongside. In all cases the additional information needed to identify a reference is provided.

The book has evolved from our experience in teaching algebra for many years at the undergraduate and graduate levels. The material has been class tested through mimeographed notes distributed to the students.

We acknowledge our indebtedness to numerous authors whose books have influenced our writing. In particular, we mention P. M. Cohn's *Algebra*, Vols. 1, 2, John Wiley, New York, 1974, 1977, and S. Lang's *Algebra*, Addison-Wesley, Reading, MA, 1965.

During the preparation of this book we received valuable help from several colleagues and graduate students. We express our gratitude to all of them. We also express our gratefulness to Ohio University for providing us the facilities to work together in the congenial environment of its beautiful campus.

It is our pleasant duty to express our gratitude to Professor Donald O. Norris, Chairman, Department of Mathematics, Ohio University, whose encouragement and unstinted support enabled us to complete our project. We also thank Mrs. Stephanie Goldsberry for the splendid job of typing.

P. B. Bhattacharya
S. K. Jain
S. R. Nagpaul

Cambridge University Press
0521460816 - Basic Abstract Algebra, Second Edition - P. B. Bhattacharya, S. K. Jain and S. R. Nagpaul
Frontmatter/Prelims
More information

Glossary of symbols

$\forall$	for all
$\exists$	there exists
$\in$	is an element of
$\notin$	is not an element of
$\{x \in A \mid P(x)\}$	set of all $x \in A$ satisfying condition $P(x)$
$\mathscr{P}(x)$	power set of x
$(X_i)_{i \in \Lambda}$	family indexed by set Λ
$\bigcup_{i \in \Lambda} X_i$	union of $(X_i)_{i \in \Lambda}$
$\bigcap_{i \in \Lambda} X_i$	intersection of $(X_i)_{i \in \Lambda}$
$X \times Y$	Cartesian product of X and Y
$\varnothing$	empty set
$\subset$ or $\subseteq$	is a subset of
$\subsetneq$	is a proper subset of
$\supset$ or $\supseteq$	contains
$\supsetneq$	properly contains
$\Rightarrow$	implies
$\Leftrightarrow$	if and only if
iff	if and only if
$f: X \to Y$	f is a map of X into Y
$f(x)$	image of $x \in X$ under $f: X \to Y$
$f: x \mapsto y$	$y = f(x)$ where $f: X \to Y$, $x \in X$, $y \in Y$
$\circ$	composition
ϕ	Euler's function

(a,b)	in number theory, the greatest common divisor of a and b; in rings and modules, the ideal or submodule generated by a and b
$a \mid b$	a divides b
$a \nmid b$	a does not divide b
δ_{ij}	Kronecker delta
det	determinant
sgn σ	± 1, according as the permutation σ is even or odd
e_{ij}	square matrix with 1 in (i, j) position, 0 elsewhere
$\mathbf{n}$	the set of positive integers $\{1,2,3,\dots,n\}$
$\mathbf{N}$	set of all natural numbers
$\mathbf{Z}$	set of all integers
$\mathbf{Q}$	set of all rational numbers
$\mathbf{R}$	set of all real numbers
$\mathbf{C}$	set of all complex numbers
$\mathfrak{c}$	the cardinal of the continuum (cardinality of the reals)
$Z/(n)$ or Z_n	integers modulo n
$\lvert X \rvert$ or card X	cardinality of X
$\lvert G \rvert$	order of group G
$[S]$	subgroup generated by S
C_n	cyclic group of order n
S_n	as a group, the symmetric group of degree n; as a ring, the ring of $n \times n$ matrices over S
A_n	alternating group of degree n
D_n	dihedral group of degree n
$\mathrm{GL}(m, F)$	group of invertible $m \times m$ matrices over F
$Z(G)$	center of G
$\triangleleft$	is a normal subgroup of
A/B	quotient group (ring, module) of A modulo B
$[L:K]$	in groups, the index of a subgroup K in a group L; in vector spaces, the dimension of a vector space L over K; in fields, the degree of extension of L over K
$N(S)$ $(N_H(S))$	normalizer of S (in H)
$C(S)$ $(C_H(S))$	conjugate class of S (with respect to H)
$\prod_{i \in \Lambda} X_i$	product of $(X_i)_{i \in \Lambda}$
$\oplus \sum_{i \in \Lambda} X_i$	direct sum of $(X_i)_{i \in \Lambda}$
$\mathrm{Im}\, f$	image of homomorphism f
$\mathrm{Ker}\, f$	kernel of homomorphism f
$\hookrightarrow$	is isomorphic into (embeddable)
$\simeq$	is isomorphic onto

R^{op}	opposite ring of R
(S)	ideal (submodule) generated by S
$(S)_{\mathrm{r}}$	right ideal generated by S
$(S)_{\mathrm{l}}$	left ideal generated by S
$\sum_{i\in\Lambda} X_i$	sum of right or left ideals (submodules) $(X_i)_{i\in\Lambda}$
$R[x]$	polynomial ring over R in one indeterminate x
$R[x_1,\dots,x_n]$	polynomial ring over R in n indeterminates, $x_1,\dots,x_n$
$R[[x]]$	formal power series ring
$R\langle x\rangle$	ring of formal Laurent series
$Z(p\infty)$	rationals between 0 and 1 of the form m/p^n, $m,n > 0$ under the binary operation "addition modulo 1"
$\mathrm{Hom}_R(X,Y)$	set of all R-homomorphisms of R-module X to R-module Y
$\mathrm{Hom}(X,Y)$	set of all homomorphisms of X to Y
$\mathrm{End}(X)$	endomorphisms of X
$\mathrm{Aut}(X)$	automorphisms of X
R_S	localization of a ring R at S
$F(\alpha)$	subfield generated by F and α
$F[S]$	subring generated by F and S
$F(S)$	subfield generated by F and S
$GF(q)$	Galois field (finite field) with q elements
$\overline{F}$	algebraic closure of F
E_H	fixed field of H
$G(E/F)$	Galois group of automorphisms of E over F
$\phi_n(x)$	cyclotomic polynomial of degree n
$M\otimes_R N$	tensor product of M_R and $_RN$
$\square$	end of the proof