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Preface

These lecture notes are intended to provide third-year mathematics undergraduates who are already familiar with inviscid fluid dynamics with some of the basic facts about the modelling and analysis of viscous flows. Writing the notes has been an interesting task because so many of the phenomena to be described are not only associated with vitally important mechanisms in everyday life but they are also readily observable without any need for instrumentation. More sophisticated realisations are also readily available, for instance in the very valuable collection of photographs “An Album of Fluid Motion”, edited by Van Dyke (2). Thus it is all the more stimulating when the mathematics that emerges when these phenomena are modelled is novel and suggestive of new methodologies.

The notes are strictly *not* self-contained and should be read in conjunction with standard texts which are referenced. We have concentrated on trying to present some of the salient physical ideas and mathematical ramifications as starkly as possible and, to this end, many of the exercises have been designed to be worked as an integral part of the notes; they are only put at the end of chapters for convenience. The starred exercises cover more advanced material and can be omitted at a first reading.

Experience has shown us that, in the twentieth century, theoretical mechanics generally has been one of the best vehicles for learning about physical applied mathematics. We hope that by showing students some of the basic theoretical framework which has developed as a result of the study of viscous flows, they will not only be able to delve further into the subject but also be well placed to exploit mathematical ideas throughout the whole of applied science.

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