

# Contents

Preface	ix
1. What is Combinatorics?	1
<i>Sample problems — How to use this book — What you need to know — Exercises</i>	
2. On numbers and counting	7
<i>Natural numbers and arithmetic — Induction — Some useful functions — Orders of magnitude — Different ways of counting — Double counting — Appendix on set notation — Exercises</i>	
3. Subsets, partitions, permutations	21
<i>Subsets — Subsets of fixed size — The Binomial Theorem and Pascal's Triangle — Project: Congruences of binomial coefficients — Permutations — Estimates for factorials — Selections — Equivalence and order — Project: Finite topologies — Project: Cayley's Theorem on trees — Bell numbers — Generating combinatorial objects — Exercises</i>	
4. Recurrence relations and generating functions	49
<i>Fibonacci numbers — Aside on formal power series — Linear recurrence relations with constant coefficients — Derangements and involutions — Catalan and Bell numbers — Computing solutions to recurrence relations — Project: Finite fields and QUICKSORT — Exercises</i>	
5. The Principle of Inclusion and Exclusion	75
<i>PIE — A generalisation — Stirling numbers — Project: Stirling numbers and exponentials — Even and odd permutations — Exercises</i>	
6. Latin squares and SDRs	87
<i>Latin squares — Systems of distinct representatives — How many Latin squares? — Quasigroups — Project: Quasigroups and groups — Orthogonal Latin squares — Exercises</i>	
7. Extremal set theory	99
<i>Intersecting families — Sperner families — The De Bruijn–Erdős Theorem — Project: Regular families — Exercises</i>	
8. Steiner triple systems	107
<i>Steiner systems — A direct construction — A recursive construction — Packing and covering — Project: Some special Steiner triple systems — Project: Tournaments and Kirkman's schoolgirls — Exercises</i>	
9. Finite geometry	123
<i>Linear algebra over finite fields — Gaussian coefficients — Projective geometry — Axioms for projective geometry — Projective planes — Other kinds of geometry — Project: Coordinates and configurations — Project: Proof of the Bruck–Ryser Theorem — Appendix: Finite fields — Exercises</i>	
10. Ramsey's Theorem	147
<i>The Pigeonhole Principle — Some special cases — Ramsey's Theorem — Bounds for Ramsey numbers — Applications — The infinite version — Exercises</i>	

viii	Contents	
11. Graphs		159
	<i>Definitions — Trees and forests — Minimal spanning trees — Eulerian graphs — Hamiltonian graphs — Project: Gray codes — The Travelling Salesman — Digraphs — Networks — Menger, König and Hall — Diameter and girth — Project: Moore graphs — Exercises</i>	
12. Posets, lattices and matroids		187
	<i>Posets and lattices — Linear extensions of a poset — Distributive lattices — Aside on propositional logic — Chains and antichains — Products and dimension — The Möbius function of a poset — Matroids — Project: Arrow's Theorem — Exercises</i>	
13. More on partitions and permutations		209
	<i>Partitions, diagrams and conjugacy classes — Euler's Pentagonal Numbers Theorem — Project: Jacobi's Identity — Tableaux — Symmetric polynomials — Exercises</i>	
14. Automorphism groups and permutation groups		225
	<i>Three definitions of a group — Examples of groups — Orbits and transitivity — The Schreier–Sims algorithm — Primitivity and multiple transitivity — Examples — Project: Cayley digraphs and Frucht's Theorem — Exercises</i>	
15. Enumeration under group action		245
	<i>The Orbit-counting Lemma — An application — Cycle index — Examples — Direct and wreath products — Stirling numbers revisited — Project: Cycle index and symmetric functions — Exercises</i>	
16. Designs		257
	<i>Definitions and examples — To repeat or not to repeat — Fisher's Inequality — Designs from finite geometry — Small designs — Project: Hadamard matrices — Exercises</i>	
17. Error-correcting codes		271
	<i>Finding out a liar — Definitions — Probabilistic considerations — Some bounds — Linear codes; Hamming codes — Perfect codes — Linear codes and projective spaces — Exercises</i>	
18. Graph colourings		291
	<i>More on bipartite graphs — Vertex colourings — Project: Brooks' Theorem — Perfect graphs — Edge colourings — Topological graph theory — Project: The Five-colour Theorem — Exercises</i>	
19. The infinite		307
	<i>Counting infinite sets — König's Infinity Lemma — Posets and Zorn's Lemma — Ramsey theory — Systems of distinct representatives — Free constructions — The random graph — Exercises</i>	
20. Where to from here?		325
	<i>Computational complexity — Some graph-theoretic topics — Computer software — Unsolved problems — Further reading</i>	
Answers to selected exercises		339
Bibliography		343
Index		347