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978-0-521-45761-3 - Combinatorics: Topics, Techniques, Algorithms

Peter J. Cameron

Frontmatter

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Combinatorics is a subject of increasing importance, owing to its links with computer science, statistics and algebra. This is a textbook aimed at second-year undergraduates to beginning graduates. It stresses common techniques (such as generating functions and recursive construction) which underlie the great variety of subject matter, and the fact that a constructive or algorithmic proof is more valuable than an existence proof.

The book is divided into two parts, the second at a higher level and with a wider range than the first. Historical notes are included and give a wider perspective on the subject. More advanced topics are given as projects, and there are a number of exercises, some with solutions given.

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COMBINATORICS: TOPICS, TECHNIQUES, ALGORITHMS

PETER J. CAMERON

Queen Mary & Westfield College, London



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Preface

I've got to work the equations and the low cations
I've got to comb the nations of it.

Russell Hoban, *Riddley Walker* (1980)

We have not begun to understand the relationship between combinatorics and
conceptual mathematics.

J. Dieudonné, *A Panorama of Pure Mathematics* (1982)

If anything at all can be deduced from the two quotations at the top of this page, perhaps it is this: Combinatorics is an essential part of the human spirit; but it is a difficult subject for the abstract, axiomatising Bourbaki school of mathematics to comprehend. Nevertheless, the advent of computers and electronic communications have made it a more important subject than ever.

This is a textbook on combinatorics. It's based on my experience of more than twenty years of research and, more specifically, on teaching a course at Queen Mary and Westfield College, University of London, since 1986. The book presupposes some mathematical knowledge. The first part (Chapters 2–11) could be studied by a second-year British undergraduate; but I hope that more advanced students will find something interesting here too (especially in the Projects, which may be skipped without much loss by beginners). The second half (Chapters 12–20) is in a more condensed style, more suited to postgraduate students.

I am grateful to many colleagues, friends and students for all kinds of contributions, some of which are acknowledged in the text; and to Neill Cameron, for the illustration on p. 128.

I have not provided a table of dependencies between chapters. Everything is connected; but combinatorics is, by nature, broad rather than deep. The more important connections are indicated at the start of the chapters.

Peter J. Cameron
17 March 1994